CS720

Logical Foundations of Computer Science

Lecture 1: course structure, Coq basics

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About the course

- **Classes:** Tuesday & Thursday
  12:30noon to 1:45pm at S-3-028
- **Office hours:** Tuesday & Thursday
  2:30pm to 4:00pm at S-3-088
- **Course web page:** `piazza.com/umb/fall2018/cs720/home`

Grading

- Homework: 75%
- Presentation: 15%
- Participation: 10%

Homework (75%)

- No late homework. Late homework $=0$ points.
- Homework can be resubmitted up to **one week**. Final grade is the average of both submissions.
- Your lowest homework score will be dropped.
- Homework is your personal individual work.

> It is *acceptable* to discuss the concept in general terms, but *unacceptable* to discuss specific solutions to any homework assignment.

Autograding

- Work is graded automatically. If Coq cannot check your homework, then your grade is 0 points.
- Use `Admitted` to allow for incomplete proofs and definitions.

Presentation (15%)

Choose one:

1. One chapter of the textbook to the class (60 minutes).
   The instructor will still publish the slides of that chapter.
2. One paper on the subject of formalizing the semantics of a programming language or system (20 minutes presentation).
   The student may suggest a paper (or request one suggestion).

Participation (10%)

Each reasonable student intervention (in the class or online) yields 1 point. If the student reaches 14 points, they are graded the full mark of participation.

Textbooks

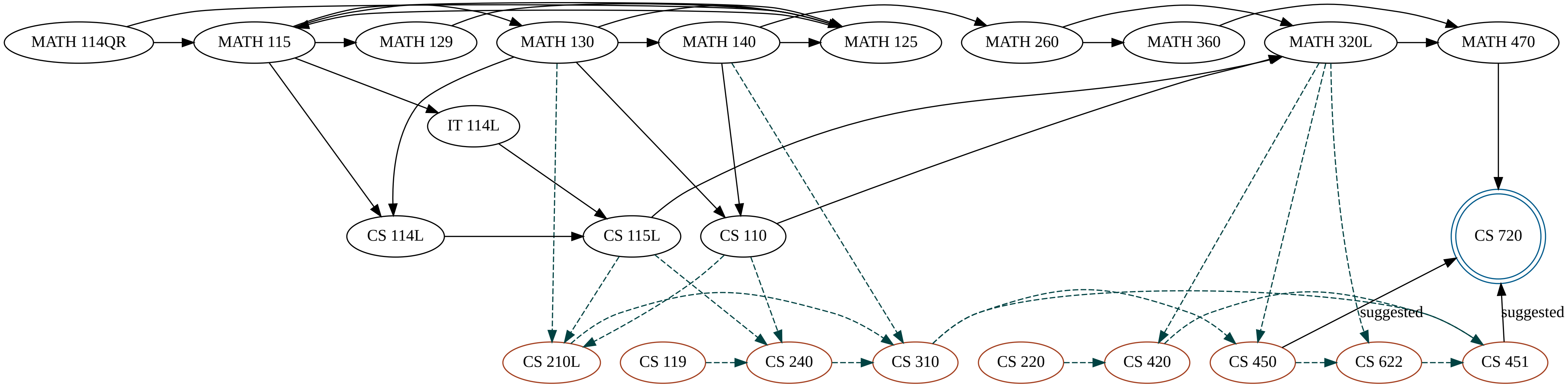
- Logical Foundations (Software Foundations - Volume 1). Benjamin C. Pierce, *et al.* 2017. Version 5.3.
- Programming Languages Foundations (Software Foundations - Volume 2). Benjamin C. Pierce,*et al.* 2017. Version 5.3.

Recommended

- Types and programming languages. Benjamin C. Pierce. 2002.
- Software foundations @ YouTube
- Oregon PL Summer School Archives (in particular: 2013, 2014,)

Suggested background

- CS 450: The Structure of Higher Level Languages
- CS 451: Compilers



Programming language semantics

- Describes a **computation model**
- Defines the set of possible behaviors through some primitives
- Mathematically precise properties of a computation model

Bird's eye view

Here is what we will learn

Imperative program

```
let division (a b: int) : int
=
  let q = ref 0 in
  let r = ref a in
  while !r ≥ b do
    q := !q + 1;
    r := !r - b
  done;
  !q
```

Examples: OCaml, F#, ReasonML

Specifying a functional language

Language grammar

$$t ::= x \mid v \mid t\; t \qquad v ::= \lambda x\colon T.t \qquad T ::= T \to T \mid \texttt{unit}$$

Evaluation rules

$$\frac{t_1 \longrightarrow t_1'}{t_1\; t_2 \longrightarrow t_1'\; t_2}\ \texttt{(E-app1)} \qquad \frac{t_2 \longrightarrow t_2'}{t_1\; t_2 \longrightarrow t_1\; t_2'}\ \texttt{(E-app2)}$$

$$(\lambda x\colon T_{11}.t_{12})\; v_2 \longrightarrow [x \mapsto v_2]t_{12}\ \texttt{(E-abs)}$$

Specifying a functional language

Type checking rules

$$\frac{\Gamma(x) = T}{\Gamma \vdash x : T} \; \texttt{(T-var)} \qquad \frac{\Gamma[x \mapsto T_1] \vdash t_2 : T_2}{\Gamma \vdash \lambda x : T_1 . t_2 : T_1 \to T_2} \; \texttt{(T-abs)}$$

$$\frac{\Gamma \vdash t_1 : T_{11} \to T_{12} \qquad \Gamma \vdash t_2 : T_{11}}{\Gamma \vdash \lambda x : T_1 . t_2 : T_1 \to T_2} \; \texttt{(T-app)}$$

Mathematically precise properties

Progress

Any valid program is either a value or can evaluate.
If $\Gamma \vdash t : T$, then either t is a value, or there exists some t' such that $t \longrightarrow t'$.

Subject reduction

The validity of a program is preserved while evaluating it.
If $\Gamma \vdash t : T$ and $t \longrightarrow t'$, then $\Gamma \vdash t' : T$.

> Can you give an example of a property?

Pre- and post-conditions

```
let division (a b: int) : int
  requires { true }
  ensures  { exists r: int. a = b * result + r /\ 0 ≤ r < b }
=
  let q = ref 0 in
  let r = ref a in
  while !r ≥ b do
    invariant { true }
    q := !q + 1;
    r := !r - b
  done;
  !q
```

Examples: WhyML, Dafny.

What we will learn in this course

Course summary

Specification: logical reasoning, describing program behavior

Abstraction: capturing the fundamentals, thinking from first principles

Testing: unit and property testing

: Part 1

A primer on the programming language Coq

We will learn the core principles behind Coq

Enumerated type

A data type where the user specifies the various distinct values that inhabit the type.

Examples?

Enumerated type

A data type where the user specifies the various distinct values that inhabit the type.

Examples?

- boolean
- 4 suits of cards
- byte
- int32
- int64

Declare an enumerated type

```
Inductive day : Type :=
  | monday : day
  | tuesday : day
  | wednesday : day
  | thursday : day
  | friday : day
  | saturday : day
  | sunday : day.
```

- `Inductive` defines an (enumerated) type by cases.
- The type is named `day` and declared as a `: Type` (Line 1).
- Enumerated types are delimited by the assignment operator (`:=`) and a dot (`.`).
- Type `day` consists of 7 cases, each of which is is tagged with the type (`day`).

Printing to the standard output

`Compute` prints the result of an expression (terminated with dot):

```
Compute monday.
```

prints

```
     = tuesday
     : day
```

Interacting with the outside world

- Programming in Coq is different most popular programming paradigms
- Programming is an **interactive** development process
- The IDE is very helpful: workflow similar to using a debugger
- It's a REPL on steroids!
- `Compute` evaluates an expression, similar to `printf`

Inspecting an enumerated type

```
match d with
| monday ⇒ tuesday
| tuesday ⇒ wednesday
| wednesday ⇒ thursday
| thursday ⇒ friday
| friday ⇒ monday
| saturday ⇒ monday
| sunday ⇒ monday
end
```

Inspecting an enumerated type

```
match d with
| monday ⇒ tuesday
| tuesday ⇒ wednesday
| wednesday ⇒ thursday
| thursday ⇒ friday
| friday ⇒ monday
| saturday ⇒ monday
| sunday ⇒ monday
end
```

- `match` performs **pattern matching** on variable `d`.
- Each pattern-match is called a *branch*; the branches are delimited by keywords `with` and `end`.
- Each *branch* is prefixed by a mid-bar (|) (⇒), a pattern (eg, `monday`), an arrow (⇒), and a return value

Pattern matching example

```
Compute match monday with
| monday ⇒ tuesday
| tuesday ⇒ wednesday
| wednesday ⇒ thursday
| thursday ⇒ friday
| friday ⇒ monday
| saturday ⇒ monday
| sunday ⇒ monday
end.
```

Create a function

```
Definition next_weekday (d:day) : day :=
  match d with
  | monday ⇒ tuesday
  | tuesday ⇒ wednesday
  | wednesday ⇒ thursday
  | thursday ⇒ friday
  | friday ⇒ monday
  | saturday ⇒ monday
  | sunday ⇒ monday
  end.
```

Create a function

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Definition next_weekday (d:day) : day :=
  match d with
  | monday ⇒ tuesday
  | tuesday ⇒ wednesday
  | wednesday ⇒ thursday
  | thursday ⇒ friday
  | friday ⇒ monday
  | saturday ⇒ monday
  | sunday ⇒ monday
  end.
```

- `Definition` is used to declare a function.
- In this case `next_weekday` has one parameter `d` of type `day` and returns (`:`) a value of type `day`.
- Between the assignment operator (`:=`) and the dot (`.`), we have the body of the function.

Example 2

```
Compute (next_weekday friday).
```

yields (Message pane)

```
     = monday
     : day
```

`next_weekday friday` is the same as `monday` (after evaluation)

Your first proof

```
Example test_next_weekday:
  next_weekday (next_weekday saturday) = tuesday.
Proof.
  simpl.       (* simplify left-hand side *)
  reflexivity. (* use reflexivity since we have tuesday = tuesday *)
Qed.
```

Your first proof

```
Example test_next_weekday:
  next_weekday (next_weekday saturday) = tuesday.
Proof.
  simpl.       (* simplify left-hand side *)
  reflexivity. (* use reflexivity since we have tuesday = tuesday *)
Qed.
```

- `Example` prefixes the name of the proposition we want to prove.
- The return type (:) is a (logical) **proposition** stating that two values are equal (after evaluation).
- The body of function `test_next_weekday` uses the `ltac` proof language.
- The dot (.) after the type puts us in proof mode. (Read as "defined below".)
- This is essentially a unit test.

Ltac: Coq's proof language

`ltac` is **imperative!** You can step through the state with CoqIDE

`Proof` begins an `ltac`-scope, yielding

```
1 subgoal
______________________________________(1/1)
next_weekday (next_weekday saturday) = tuesday
```

Tactic `simpl` evaluates expressions in a goal (normalizes them)

Ltac: Coq's proof language

```
1 subgoal
______________________________________(1/1)
tuesday = tuesday
```

- `reflexivity` solves a goal with a pattern `?X = ?X`

```
No more subgoals.
```

- `Qed` ends an `ltac`-scope and ensures nothing is left to prove

Function types

Use `Check` to print the type of an expression:

```
Check next_weekday.
```

which outputs

```
next_weekday
     : day -> day
```

Function type `day -> day` takes one value of type `day` and returns a value of type `day`.

Compound types

Enumerated types are very simple. You can think of them as a typed collection of constants. We call each enumerated value a **constructor**.

```
Inductive rgb : Type :=
  | red : rgb
  | green : rgb
  | blue : rgb.
```

Compound types

Enumerated types are very simple. You can think of them as a typed collection of constants. We call each enumerated value a **constructor**.

```
Inductive rgb : Type :=
  | red : rgb
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  | blue : rgb.
```

A **compound type** builds on other existing types. Their constructors accept *multiple parameters*, like functions do.

```
Inductive color : Type :=
  | black : color
  | white : color
  | primary : rgb → color.
```

Manipulating compound values

```
Definition monochrome (c : color) : bool :=
  match c with
  | black => true
  | white => true
  | primary p => false
  end.
```

Manipulating compound values

```
Definition monochrome (c : color) : bool :=
  match c with
  | black ⇒ true
  | white ⇒ true
  | primary p ⇒ false
  end.
```

We can use the place-holder keyword _ to mean a variable we do not mean to use.

```
Definition monochrome (c : color) : bool :=
  match c with
  | black ⇒ true
  | white ⇒ true
  | primary _ ⇒ false
  end.
```

Compound types

Allows you to: type-tag, fixed-number of values

Inductive types

How do we describe arbitrarily large/composed values?

Inductive types

How do we describe arbitrarily large/composed values?

Here's the definition of natural numbers, as found in the standard library:

```
Inductive nat : Type :=
  | O : nat
  | S : nat → nat.
```

- `O` is a constructor of type `nat`.
  Think of the numeral `0`.

- If `n` is an expression of type `nat`, then `S n` is also an expression of type `nat`.
  Think of expression `n + 1`.

> What's the difference between `nat` and `uint32`?

Recursive functions

Recursive functions are declared differently with `Fixpoint`, rather than `Definition`.

```
Fixpoint evenb (n:nat) : bool :=
  match n with
  | O ⇒ true
  | S O ⇒ false
  | S (S n') ⇒ evenb n'
  end.
```

Using `Definition` instead of `Fixpoint` will throw the following error:

```
The reference evenb was not found in the current environment.
```

Not all recursive functions can be described. Coq has to understand that one value is getting "smaller."

All functions must be total: all inputs must produce one output. *All functions must terminate.*

Basic.v

- New syntax: `Definition` declares a non-recursive function
- New syntax: `Compute` evaluates an expression and outputs the result + type
- New syntax: `Check` prints the type of an expression
- New syntax: `Inductive` defines inductive data structures
- New syntax: `Fixpoint` declares a (possibly) recursive function
- New syntax: `match` performs pattern matching on a value
- New tactic: `simpl` evaluates functions if possible
- New tactic: `reflexivity` concludes a goal `?X = ?X`

Ltac vocabulary

- `simpl`
- `reflexivity`