CS450
Structure of Higher Level Languages
Lecture 13: Reduction
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Today we will learn...

1. Revise general recursion patterns
2. Implement general recursion patterns
3. Refactor code to reduce code repetition
4. Refactor code to improve performance
Functional patterns:
Reduction
Appending two lists together

Implement function (append 11 12) that appends two lists together.

Spec

(check-equal? (append (list 1 2) (list 3 4)) (list 1 2 3 4))
Appending two lists together

Implement function \((\text{append} \; \text{l1} \; \text{l2})\) that appends two lists together.

Spec

\[
\begin{align*}
\text{(check-equal?} & \text{(append (list 1 2) (list 3 4))} \\
& \text{(list 1 2 3 4))}
\end{align*}
\]

Solution

\[
\begin{align*}
\text{(define (append l1 l2)} & \text{)} \\
& \text{(cond [(empty? l1) l2] [else (cons (first l1) (append (rest l1) l2))])}
\end{align*}
\]

Is it tail recursive?
Appending two lists together

Implement function \( (\text{append} \ l1 \ l2) \) that appends two lists together.

Spec

\[
(\text{check-equal?}\
 \quad (\text{append} \ (\text{list} \ 1 \ 2) \ (\text{list} \ 3 \ 4))\
(\text{list} \ 1 \ 2 \ 3 \ 4))
\]

Solution

\[
(\text{define} \ (\text{append} \ l1 \ l2)\
 \quad (\text{cond} \ [(\text{empty?} \ l1) \ l2]\
 \quad \quad [\text{else} \ (\text{cons} \ (\text{first} \ l1) \ (\text{append} \ (\text{rest} \ l1) \ l2))])
)\]

Is it tail recursive? \textbf{No!}
A pattern arises

; Example 1:
(define (map f l)
  (cond [(empty? l) empty]
        [else (cons (f (first l))
                   (map f (rest l)))]))

; Example 2:
(define (filter to-keep? l)
  (cond [(empty? l) empty]
        [else (cond [(to-keep? (first l))
                     (cons (first l)
                           (filter to-keep? (rest l)))]
                [else (filter to-keep? (rest l)))]))

; Example 3:
(define (append l1 l2)
  (cond [(empty? l1) l2]
        [else (cons (first l1)
                    (append (rest l1) l2))])))
A pattern arises

; Example 1:
(define (map f l)
  (cond [(empty? l) empty]
        [else (cons (f (first l)) (map f (rest l)))]))

; Example 2:
(define (filter to-keep? l)
  (cond [(empty? l) empty]
        [else (cond [(to-keep? (first l)) (cons (first l) (filter to-keep? (rest l)))]
                      [else (filter to-keep? (rest l))])])

; Example 3:
(define (append l1 l2)
  (cond [(empty? l1) l2]
        [else (cons (first l1) (append (rest l1) l2))]))

General recursion pattern for lists

(define (rec l)
  (cond [(empty? l) base-case]
        [else (step (first l) (rec (rest l)))]))

For instance,

(cons (f (first l)) (map f (rest l)))
/maps to

(step (first l) (rec (rest l)))
Implementing this recursion pattern

Recursive pattern for lists

\[
\text{define} \quad (\text{rec} \ l) \\
\quad \text{(cond)} \\
\quad \quad [\text{(empty? } l\text{)} \rightarrow \text{base-case}] \\
\quad \quad \text{[else} \quad (\text{step} \ (\text{first} \ l) \\
\quad \quad \quad \quad (\text{rec} \ (\text{rest} \ l)))]
\]

Fold right reduction

\[
\text{define} \quad (\text{foldr} \ \text{step} \ \text{base-case} \ l) \\
\quad \text{(cond)} \\
\quad \quad [\text{(empty? } l\text{)} \rightarrow \text{base-case}] \\
\quad \quad \text{[else} \quad (\text{step} \ (\text{first} \ l) \\
\quad \quad \quad \quad (\text{foldr} \ \text{step} \ \text{base-case} \ (\text{rest} \ l)))]
\]
Implementing map with foldr

```
(define (map f l)
  (cond [(empty? l) empty]
        [else (cons (f (first l))
                    (map f (rest l)))]))
```
Implementing map with foldr

\[
\text{(define (map } f \ l) \text{)}
\begin{cases}
\text{empty} & \text{(empty } l) \\
\text{else (cons } (f \text{ (first } l)) \text{ (map } f \text{ (rest } l))) & \text{else}
\end{cases}
\]

Solution

\[
\text{(define (map } f \ l) \text{)}
\text{(define (on-elem } elem \text{ new-list)}
\begin{cases}
\text{cons } (f \text{ elem) new-list}) & \text{(cons (f elem) new-list))}
\end{cases}
\text{(foldr on-elem empty } l))
\]
Implementing append with foldr

```
(define (append l1 l2)
  (cond [(empty? l1) l2]
        [else (cons (first l1) (append (rest l1) l2))]))
```
Implementing append with foldr

(define (append l1 l2)
  (cond [(empty? l1) l2]
       [else (cons (first l1)
                    (append (rest l1) l2))]))

Solution

(define (append l1 l2)
  (foldr cons l2 l1))
Implementing filter with foldr

\[
\text{(define (filter to-keep? l)}
\text{(cond}
\text{[ (empty? l) empty]}
\text{[else}
\text{ (cond [(to-keep? (first l))}
\text{ (cons (first l)}
\text{ (filter to-keep? (rest l)))]}
\text{[else (filter to-keep? (rest l)))]]))}
\]

Solution

\[
\text{(define (filter to-keep? l)}
\text{(define (on-elem elem new-list)}
\text{(cond [(to-keep? elem) (cons elem new-list)]}
\text{[else new-list]]))}
\text{(foldr on-elem empty l))}
\]
Contrasting the effect of using foldr

; Example 1:
(define (map f l)
  (cond [(empty? l) empty]
        [else (cons (f (first l))
                   (map f (rest l)))]))

; Example 2:
(define (filter to-keep? l)
  (cond [(empty? l) empty]
        [else (cond [(to-keep? (first l))
                     (cons (first l)
                           (filter to-keep? (rest l)))]
                 [else (filter to-keep? (rest l)))]))

; Example 3:
(define (append l1 l2)
  (cond [(empty? l1) l2]
        [else (cons (first l1)
                   (append (rest l1) l2)]))
Contrasting the effect of using foldr

; Example 1:
(define (map f l)
  (cond [(empty? l) empty]
        [else (cons (f (first l))
                    (map f (rest l)))]))

; Example 2:
(define (filter to-keep? l)
  (define (on-elem elem new-list)
    (cond [(to-keep? elem) (cons elem new-list)]
          [else new-list]))
  (foldr on-elem empty l))

; Example 3:
(define (append l r)
  (foldr cons r l))
What about the fold left reduction?
Reversing a list

Implement function `(reverse 1)` that reverses a list.

Spec

```scheme
(check-equal? (list 4 3 2 1) (reverse (list 1 2 3 4)))
```
Reversing a list

Implement function (reverse 1) that reverses a list.

Spec

```
(check-equal? (list 4 3 2 1) (reverse (list 1 2 3 4)))
```

Solution

```
(define (reverse 1)
  (define (rev 1 accum)
    (cond [(empty? 1) accum]
          [else (rev (rest 1) (cons (first 1) accum))])
  (rev 1 empty))
```
Another pattern arises

A generalized recursion pattern for lists

```
(define (rec base-case l)
  (cond
    [(empty? l) base-case]
    [else
      (rec (step (first l) base-case)
           (rest l))]))
```

For instance,

```
(define (first l) (first l))
(define (rest l) (rest l))
```

maps to

```
(define (step l) (step l))
```

---

; Example 1
(define (concat-nums 1)
  (define (f n)
    (string-append " " (number->string n)))
  (define (concat-nums-aux accum l)
    (cond
      [(empty? l) accum]
      [else
       (concat-nums-aux
         (string-append accum (f (first l))
          (rest l)))]))
  (concat-nums-aux "" 1))

; Example 2
(define (reverse 1)
  (define (rev accum l)
    (cond
      [(empty? l) accum]
      [else (rev (cons (first l) accum) (rest l))]))
  (rev empty 1))
```
Implementing this recursion pattern

Recursive pattern for lists

\[
\text{(define (rec accum l)} \\text{(cond}
  \text{[(empty? l) accum]}
  \text{[else}
    \text{(rec (step (first l) accum) (rest l))])})
\]

Fold left reduction

\[
\text{(define (foldl step base-case l)} \\text{(cond}
  \text{[(empty? l) base-case]}
  \text{[else (foldl step}
    \text{(step (first l) base-case) (rest l))])})
\]
Implementing concat-nums with foldl

Before

```
(define (concat-nums l)
  (define (f n)
    (string-append " " (number->string n)))
  (define (concat-nums-aux accum l)
    (cond
      [(empty? l) accum]
      [else
        (concat-nums-aux
          (string-append accum (f (first l)))
          (rest l))])
    (concat-nums-aux ">" l))
```
Implementing concat-nums with foldl

Before

```
(define (concat-nums l)
  (define (f n)
    (string-append " " (number->string n)))
  (define (concat-nums-aux accum l)
    (cond
      [(empty? l) accum]
      [else
        (concat-nums-aux
          (string-append accum (f (first l)))
          (rest l))]])
  (concat-nums-aux "">" l))
```

After

```
(define (concat-nums l)
  (define (f n a)
    (string-append a " "
      (number->string n)))
  (foldl f "">" l))
```
Implementing reverse with foldl

Original

```
(define (reverse l)
  (define (rev accum l)
    (cond [(empty? l) accum]
          [else (rev (cons (first l) accum)
                     (rest l))]]
    (rev empty l))
```

Implementing reverse with foldl

Original

```scheme
(define (reverse l)
  (define (rev accum l)
    (cond [(empty? l) accum]
          [else (rev (cons (first l) accum) (rest l))]))
  (rev empty l))
```

Solution

```scheme
(define (reverse l)
  (foldl cons empty l))
```
Contrasting the effect of using foldl

Before

; Example 1
(define (concat-nums l)
  (define (f n)
    (string-append " " (number->string n)))
  (define (concat-nums-aux accum l)
    (cond
      [(empty? l) accum]
      [else
       (concat-nums-aux
         (string-append accum (f (first l)))
         (rest l))]
    (concat-nums-aux "" 1))

; Example 2
(define (reverse l)
  (define (rev accum 1)
    (cond [(empty? l) accum]
      [else (rev (cons (first l) accum)
                            (rest l))]
    (rev empty l))

Contrasting the effect of using foldl

Before
Contrasting the effect of using foldl

Before

; Example 1
(define (concat-nums l)
  (define (f n)
    (string-append " " (number->string n)))
  (define (concat-nums-aux accum l)
    (cond
      [(empty? l) accum]
      [else
       (concat-nums-aux
        (string-append accum (f (first l)))
        (rest l))])))
  (concat-nums-aux "" l))

; Example 2
(define (reverse l)
  (define (rev accum l)
    (cond
      [(empty? l) accum]
      [else (rev (cons (first l) accum)
                (rest l))]))
  (rev empty l))

After

; Example 1
(define (concat-nums l)
  (define (f n a)
    (string-append a " "
      (number->string n)))
  (foldl f "" l))

; Example 2
(define (reverse l)
  (foldl cons empty l))
What about tail-recursive optimization?
What about tail-recursive optimization?

- We note that \texttt{foldl} is tail-recursive already
- However, our original implementation of \texttt{foldr} is not tail recursive

Can't we implement the tail-recursive optimization pattern?

Unoptimized

\begin{verbatim}
(define (rec 1)
  (cond
    [(empty? l) base-case]
    [else (step (first l) (rec (rest l)))]))
\end{verbatim}

Optimized

\begin{verbatim}
(define (rec 1)
  (define (rec-aux accum 1)
    (cond
      [(empty? 1) (accum base-case)]
      [else (rec-aux
        (lambda (x)
          (accum (step (first 1) x)))
        (rest 1)))]))
\end{verbatim}
Optimized foldr

Generalized pattern

```
(define (rec l)
  (define (rec-aux accum l)
    (cond
      [(empty? l) (accum base-case)]
      [else
       (rec-aux
         (lambda (x)
           (accum (step (first l) x)))
         (rest l))]))
    (rec-aux (lambda (x) x) 1))
```

Implementation

```
(define (foldr step base-case l)
  (define (foldr-iter accum l)
    (cond
      [(empty? l) (accum base-case)]
      [else
       (foldr-iter
         (lambda (x)
           (accum (step (first l) x)))
         (rest l))]))
    (foldr-iter (lambda (x) x) 1))
```
Benchmark evaluation

- Unoptimized foldr
- Tail-recursive foldr

Processing a list of size: 1000000
-------------------------------------------------------
Throughoutput (unopt): 7310 elems/ms
Mean (unopt): 136.8±7.56ms
-------------------------------------------------------
Throughoutput (tailrec): 12349 elems/ms
Mean (tailrec): 80.98±1.49ms
Speed-up (tailrec): 1.7

A speed improvement of 1.7
What if we use foldl + reverse?
What if we use foldl + reverse?

- Instead of creating nested functions,
- We reverse the list and apply foldl

```
(define (foldr step base-case l)
  (foldl step base-case (reverse l)))
```
What if we use foldl + reverse?

- Instead of creating nested functions,
- We reverse the list and apply foldl

```
(define (foldr step base-case l)
  (foldl step base-case (reverse l)))
```

Simpler implementation!

But is it faster?
Rev+fold runs the slower (0.7)

Processing a list of size: 1000000

Throughoutput (unopt): 7310 elems/ms
Mean (unopt): 136.8±7.56ms

Throughoutput (tailrec): 12349 elems/ms
Mean (tailrec): 80.98±1.49ms
Speed-up (tailrec): 1.7

Throughoutput (rev+foldl): 4846 elems/ms
Mean (rev+foldl): 206.34±3.33ms
Speed-up (rev+foldl): 0.7
Conclusion

We learned to generalize two reduction patterns (foldl and foldr)

- **Pro**: generalizing code can lead to a central point to optimize code
- **Pro**: generalizing code can reduce our code base
  (less code means less code to maintain)
- **Con**: one level of indirection increases the cognitive code
  (more cognitive load, code harder to understand)

Easier to understand (self-contained)

```
(define (map f l)
  (cond [(empty? l) empty]
        [else (cons (f (first l))
                    (map f (rest l)))]))
```

Harder to understand (what is foldr?)

```
(define (map f l)
  (define (on-elem elem new-list)
    (cons (f elem) new-list))
  (foldr on-elem empty l))
```