CS450

Structure of Higher Level Languages

Lecture 4: Pairs and lists

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Today we will learn...

- data structures as constructors and accessors
- pairs
- lists
- user-data structures

Function definition

Racket introduces a shorthand notation for defining functions.

```
( define (variable+ ) term+ )
```

A function definition expects one or more variables (symbols). The first variable is the function variable. The remaining variables are the arguments of the function declaration. The one-or-more terms consist of the body of the function declaration.

Which is a short-hand for:

```
( define variable (lambda ( variable* ) term+ ))
```

Exercise

The McCarthy 91 function was invented by computer scientist John McCarthy to motivate formal verification.

$$M(n) = n - 10 \text{ if } n > 100$$
$$M(n) = M(M(n + 11)) \text{ if } n \leq 100$$

- Implement the function in Racket
- What is $M(99)$?

Exercise

The McCarthy 91 function was invented by computer scientist John McCarthy to motivate formal verification.

$$M(n) = n - 10 \text{ if } n > 100$$
$$M(n) = M(M(n + 11)) \text{ if } n \leq 100$$

- Implement the function in Racket
- What is $M(99)$?

The McCarthy 91 function is equivalent to

$$M(n) = n - 10 \quad \text{if } n > 100$$
$$M(n) = 91 \quad \text{if } n \leq 100$$

Data structures

Data structures

When presenting each data structure we will introduce two sets of functions:

- **Constructors**: functions needed to build the data structure
- **Accessors**: functions needed to retrieve each component of the data structure. Also known as **selectors**.

> Each example we discuss is prefaced by some unit tests. We are following a Test Driven Development methodology.

Pairs

The pair datatype

Constructor: cons

```
expression = ··· | pair
pair = (cons expression expression )
```

Function cons constructs a pair with the evaluation of the arguments, which Racket prints as: `'(v1 . v2)`

Example

```
#lang racket
(cons (+ 1 2) (* 2 3))
```

Output

```
$ racket pair.rkt
'(3 . 6)
```

The pair datatype

Accessors: car and cdr

- Function car returns the left-hand-side element (the first element) of the pair.
- Function cdr returns the right-hand-side element (the second element) of the pair.

Example

```
#lang racket
(define pair (cons (+ 1 2) (* 2 3)))
(car pair)
(cdr pair)
```

```
$ racket pair.rkt
3
6
```

Pairs: example 1

Swap the elements of a pair: `(pair-swap p)`

Spec

```
; Paste this at the end of "pairs.rkt"
(require rackunit)
(check-equal?
  (cons 2 1)
  (pair-swap (cons 1 2)))
```

Pairs: example 1

Swap the elements of a pair: `(pair-swap p)`

Spec

```
; Paste this at the end of "pairs.rkt"
(require rackunit)
(check-equal?
  (cons 2 1)
  (pair-swap (cons 1 2)))
```

Solution

```
#lang racket
(define (pair-swap p)
  (cons
    (cdr p)
    (car p)))
```

Pairs: example 2

Point-wise addition of two pairs: `(pair+ l r)`

Unit test

```
(require rackunit)
(check-equal?
  (cons 4 6)
  (pair+ (cons 1 2) (cons 3 4)))
```

Pairs: example 2

Point-wise addition of two pairs: `(pair+ l r)`

Unit test

```
(require rackunit)
(check-equal?
  (cons 4 6)
  (pair+ (cons 1 2) (cons 3 4)))
```

Solution

```
#lang racket
(define (pair+ l r)
  (cons (+ (car l) (car r))
        (+ (cdr l) (cdr r))))
```

Pairs: example 3

Lexicographical ordering of a pair

```
(require rackunit)
(check-true (pair< (cons 1 3) (cons 2 3)))
(check-true (pair< (cons 1 2) (cons 1 3)))
(check-false (pair< (cons 1 3) (cons 1 3)))
(check-false (pair< (cons 1 3) (cons 1 0)))
```

Pairs: example 3

Lexicographical ordering of a pair

```
(require rackunit)
(check-true (pair< (cons 1 3) (cons 2 3)))
(check-true (pair< (cons 1 2) (cons 1 3)))
(check-false (pair< (cons 1 3) (cons 1 3)))
(check-false (pair< (cons 1 3) (cons 1 0)))
```

```
#lang racket
(define (pair< l r)
  (or (< (car l) (car r))
      (and (= (car l) (car r))
           (< (cdr l) (cdr r)))))
```

Lists

Lists

Constructor: list

expression = ··· | *list*
list = `(list` *expression** `)`

Function call `list` constructs a list with the evaluation of a possibly-empty sequence of expressions `e1` up to `en` as values `v1` up to `vn` which Racket prints as: `'(v1 ... v2)`

```
#lang racket
(list (+ 0 1) (+ 0 1 2) (+ 0 1 2 3))
(list)
```

```
$ racket list-ex1.rkt
'(1 3 6)
'()
```

Accessing lists

Accessor: empty?

You can test if a list is empty with function `empty?`. An empty list is printed as `'()`.

```
#lang racket
(require rackunit)
(check-false (empty? (list (+ 0 1) (+ 0 1 2) (+ 0 1 2 3))))
(check-true (empty? (list)))
```

Lists are linked-lists of pairs

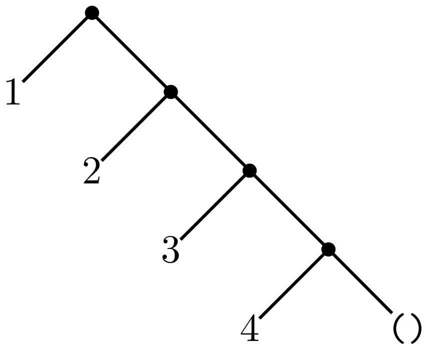
Accessors: car, cdr

Lists in Racket are implemented as a linked-list using pairs terminated by the empty list '().

- Function car returns the head of the list, given a nonempty list.
  car originally meant Contents of Address Register.
- Function cdr returns the tail of the list, given a nonempty list.
  cdr originally meant Contents of Decrement Register.

```
(list 1 2 3 4)
```

Graphical representation



Textual representation

```
'(1 .
  '(2 .
    '(3 .
      '(4 . '()))))
```

Lists are built from pairs example

Constructor `empty`

```
#lang racket
(require rackunit)
(check-equal?
  (cons 1
    (cons 2
      (cons 3
        (cons 4 empty)))) (list 1 2 3 4))
```