CS450
Structure of Higher Level Languages
Lecture 3: Functions
Tiago Cogumbreiro
The homework template is in the Forum, under the resources page:

```
(define ex1 'todo)
(define ex2 'todo)
(define ex3 'todo)
;; ...
```
Sanity check (0.0/1.0)

Are you using the homework template?
I could not find the following definitions:
* define-basic?
* define-func?
* define?
* apply-args
* apply-func
* apply?
* \lambda-body
* lambda-params
* lambda?
* bst-insert
* tree-set-value
* tree-set-right
* tree-set-left
* tree-value
* tree-right
* tree-left
* tree-leaf
* tree
* ex3
* ex2
* ex1

Tip #1: try assigning a dummy value to each definition. For instance: (define define-basic? #f)

Tip #2: ensure your definitions are made public. The first two lines of your file should be:
#lang racket
(provide (all-defined-out))
<table>
<thead>
<tr>
<th>Exercise</th>
<th>Score (Max)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.a</td>
<td>0.0/1.5</td>
</tr>
<tr>
<td>1.b</td>
<td>0.0/3.5</td>
</tr>
<tr>
<td>2</td>
<td>0.0/2.0</td>
</tr>
<tr>
<td>3. bst-insert</td>
<td>0.0/3.0</td>
</tr>
<tr>
<td>3. tree</td>
<td>0.0/0.5</td>
</tr>
<tr>
<td>3. tree-leaf</td>
<td>0.0/0.5</td>
</tr>
<tr>
<td>3. tree-left</td>
<td>0.0/0.5</td>
</tr>
<tr>
<td>3. tree-right</td>
<td>0.0/0.5</td>
</tr>
<tr>
<td>3. tree-set-left</td>
<td>0.0/0.5</td>
</tr>
<tr>
<td>3. tree-set-right</td>
<td>0.0/0.5</td>
</tr>
<tr>
<td>3. tree-set-value</td>
<td>0.0/0.5</td>
</tr>
<tr>
<td>3. tree-value</td>
<td>0.0/0.5</td>
</tr>
<tr>
<td>3. lambda?</td>
<td>0.0/0.5</td>
</tr>
<tr>
<td>3. lambda-params</td>
<td>0.0/0.5</td>
</tr>
<tr>
<td>4. c. lambda-body</td>
<td>0.0/0.5</td>
</tr>
<tr>
<td>4. d. apply?</td>
<td>0.0/0.5</td>
</tr>
<tr>
<td>4. e. apply-func, 4.f. apply-args</td>
<td>0.0/1.0</td>
</tr>
<tr>
<td>4. g. define?</td>
<td>0.0/0.5</td>
</tr>
<tr>
<td>4. h. define-base?</td>
<td>0.0/0.5</td>
</tr>
<tr>
<td>4. i. define-func?</td>
<td>0.0/0.5</td>
</tr>
</tbody>
</table>

**Score:** 0.0 / 24.0

**Failed Tests:**
- Exercise 1.a (0.0/1.5)
- Exercise 1.b (0.0/3.5)
- Exercise 2 (0.0/2.0)
- Exercise 3. bst-insert (0.0/3.0)
- Exercise 3. tree (0.0/0.5)
- Exercise 3. tree-leaf (0.0/0.5)
- Exercise 3. tree-left (0.0/0.5)
- Exercise 3. tree-right (0.0/0.5)
- Exercise 3. tree-set-left (0.0/0.5)
- Exercise 3. tree-set-right (0.0/0.5)
- Exercise 3. tree-set-value (0.0/0.5)
- Exercise 3. tree-value (0.0/0.5)
- Exercise 4. c. lambda-body (0.0/0.5)
- Exercise 4. d. apply? (0.0/1.0)
- Exercise 4. e. apply-func, 4.f. apply-args (0.0/1.0)
- Exercise 4. g. define? (0.0/0.2)
- Exercise 4. h. define-base? (0.0/1.0)
- Exercise 4. i. define-func? (0.0/2.8)
Today we will learn about...

- function declaration
- function definition

Cover up until Section 2.2.1 of the SICP book. Try out the interactive version of section 2.1 of the SICP book.
Exercise: conditionals

- The modulo operator (%) in Racket is function modulo
- The equality operator in Racket is equal?

Translate the following code to Racket:

```racket
n = 16
if n % 15 == 0:
    return "fizzbuzz"
if n % 3 == 0:
    return "fizz"
if n % 5 == 0:
    return "buzz"
return n
```
Exercise: conditionals (solution)

```
(define n 16)
(cond [(equal? (modulo n 15) 0) "fizzbuzz"
      [(equal? (modulo n 3) 0) "fizz"
       [(equal? (modulo n 5) 0) "buzz"
        [else n]])
```
Exercise: evaluation

```scheme
(define x (* 10 2))
(+ x (* 4 2))
```

Solution
Exercise: evaluation

```lisp
(define x (* 10 2))
(+ x (* 4 2))
```

Solution

5 steps

```lisp
; Step 1: eval *
(define x 20)
(+ x (* 4 2))
```
Exercise: evaluation

```scheme
(define x (* 10 2))
(+ x (* 4 2))
```

Solution

5 steps

; Step 1: `eval *`
(define x 20)
(+ x (* 4 2))

; Step 2: `eval define`
; x = 20
(+ x (* 4 2))
Exercise: evaluation

```
(define x (* 10 2))
(+ x (* 4 2))
```

Solution

5 steps

1. **Step 1:** eval *
   ```
   (define x 20)
   (+ x (* 4 2))
   ```

2. **Step 2:** eval define
   ```
   ; x = 20
   (+ x (* 4 2))
   ```

3. **Step 3:** eval x
   ```
   ; x = 20
   (+ 20 (* 4 2))
   ```
Exercise: evaluation

#define x (* 10 2)
(+ x (* 4 2))

Solution

5 steps

; Step 1: eval *
(define x 20)
(+ x (* 4 2))

; Step 2: eval define
; x = 20
(+ x (* 4 2))

; Step 3: eval x
; x = 20
(+ 20 (* 4 2))
Exercise: evaluation

// How many evaluation steps should we expect?
(define x (* 10 2))
(+ x (* 4 2))

Solution

5 steps

; Step 1: eval *
(define x 20)
(+ x (* 4 2))

; Step 4: eval *
(+ 20 8)

; Step 2: eval define
; x = 20
(+ x (* 4 2))

; Step 5: eval +
; 28

; Step 3: eval x
; x = 20
(+ 20 (* 4 2))
Variables evaluate?

Variables are considered expressions, so the runtime must lookup the value \textit{bound} to a variable as one step of the evaluation.

\begin{verbatim}
program = #lang racket expression*
expression = value | variable | function-call | ...
value = number | ...
function-call = ( expression+ )
\end{verbatim}
Function declaration
Function declaration

A function declaration creates an anonymous function and consists of:

- **parameters**: zero or more parameters (identifiers, known as symbols)
- **body** which consist of one or more terms

When calling a function we replace each argument by the parameter defined in the lambda. If the number of parameters is not the expected one, then we get an error. The return value of the function corresponds to the evaluation of the *last* term in the body (known as the **tail position**).

```
function-dec = ( lambda ( variable* ) term+)
```

We can define `circumference` as a function and parameterize the radius:

```
#lang racket
(define circumference (lambda (radius) (* 2 3.14159 radius)))
(circumference 2)
```

```
$ racket func.rkt
12.56636
```
Evaluating a lambda

\[
\text{(define circ}
  \text{ (lambda (radius) (* 2 3.14159 radius)))}
\text{ (circ 2))}
\]

\[
\text{; circ = lambda ...}
\text{ ; Prints #<void>}
\text{ (circ 2)}
\]

\[
\text{; circ = lambda ...}
\text{ ((lambda (radius) (* 2 3.14159 radius)) 2)}
\text{ ; Subst circ}
\]

\[
\text{; circ = lambda ...}
\text{ (* 2 3.14159 2)}
\text{ ; Applied func}
\]

\[
\text{; circ = lambda ...}
\text{ 12.56636}
\text{ ; Prints 12.56636}
\]

For more information on evaluation, read Section 1.1.5 of SICP.
Function definition

Racket introduces a shorthand notation for defining functions.

\[
(\text{define} \ (\text{variable}+) \ (\text{term}+))
\]

A function definition expects one or more variables (symbols). The first variable is the function variable. The remaining variables are the arguments of the function declaration. The one-or-more terms consist of the body of the function declaration. Which is a short-hand for:

\[
(\text{define} \ \text{variable} \ (\text{lambda} \ (\text{variable}*+) \ (\text{term}+)))
\]
The McCarthy 91 function was invented by computer scientist John McCarthy to motivate formal verification.

\[
M(n) = \begin{cases} 
  n - 10 & \text{if } n > 100 \\
  M(M(n + 11)) & \text{if } n \leq 100
\end{cases}
\]

- Implement the function in Racket
- What is \( M(99) \)?
Exercise

The McCarthy 91 function was invented by computer scientist John McCarthy to motivate formal verification.

\[
M(n) = n - 10 \text{ if } n > 100 \\
M(n) = M(M(n + 11)) \text{ if } n \leq 100
\]

- Implement the function in Racket
- What is \(M(99)\)?

The McCarthy 91 function is equivalent to

\[
M(n) = n - 10 \text{ if } n > 100 \\
M(n) = 91 \text{ if } n \leq 100
\]