CS420

Introduction to the Theory of Computation

Lecture M3: Module 3 Recap

Tiago Cogumbreiro

Today we will...

- Announcements
- Recap what we learned in CS 420
- Discuss what work/didn't work in CS 420
- Go over sample exercises for mini-test 3
- Course evaluation

Announcements

Mini Test 3

Where: Y02-2330, 2nd University Hall

When: from 5:30pm until 6:45pm

CSM Undergraduate Research Fellowships

Want to do research?

CSM students applying for this Fellowship need to:

- Identify a potential topic of research and a potential research group.
- Demonstrate an excellent academic standing with a minimum grade point average (GPA) of 3.2.
- Commit to working with a research group the equivalent time of 2-3 credits of coursework per semester.
- Agree to present a research poster at the CSM Student Success Showcase on Friday, May 15, 2020.

Deadline: January 15, 2020

`forms.umb.edu/csm-opportunities/c/urf`

CS 420

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  - multiple abstractions to handle the same concept and solve different problems (DFA/NFA/REGEX)

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  - Reducibility: mapping problems into other problems
  - Logic programming using a proof assistant

What work/didn't work in CS 420?

CS 420

Do you think using a proof assistant helped you?

CS 420

Do you think we should devote ***more*** time learning to use a proof assistant?

Mini Test 3 Primer

Mini Test 3 overview

- 50 points for Sections 4.1 and 4.2 (HW7 + Exercises in Lesson 20)
- around 10 points for Section 5.1
- around 40 points for Section 5.3

- Level 1: 60 points
- Level 2: 25 points
- Level 3: 15 points

Exercise 1 (Level 1)

- Know why membership tests fail and succeed; explain **why** certain membership fails.

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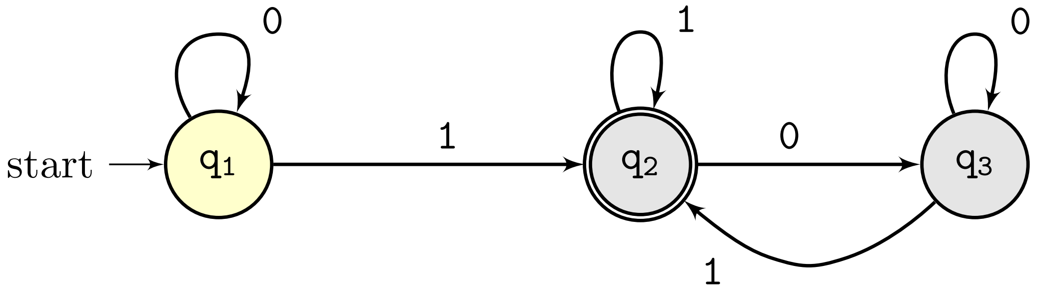
Let D be the DFA below




```
def A_DFA(D, w): return D accept w
def E_DFA(D): return L(D) == {}
def EQ_DFA(D1, D2): return L(D1) == L(D2)
```

- Exercise 2.1: Is $\langle D, 0100\rangle \in A_{DFA}$?
- Exercise 2.2: Is $\langle D, 101\rangle \in A_{DFA}$?
- Exercise 2.3: Is $\langle D\rangle \in A_{DFA}$?
- Exercise 2.4: Is $\langle D, 101\rangle \in A_{REX}$?
- Exercise 2.5: Is $\langle D\rangle \in E_{DFA}$?
- Exercise 2.6: Is $\langle D, D\rangle \in EQ_{DFA}$?
- Exercise 2.7: Is $101 \in A_{REX}$?

Exercise 2 (Level 1)

- Know how to compose decidable algorithms as new decidable algorithms.

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Give an algorithm that decides EQ_{REX}

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Give an algorithm that decides EQ_{REX}

```
def EQ_REX(R1, R2):
  return EQ_DFA(REX_TO_DFA(R1), REX_TO_DFA(R2))
```

Similar examples: give a decider for

- $A_{\text{NFA}}, A_{\text{REX}}, A_{\text{PDA}}$ (Lesson 17)
- EQ_{DFA} (Lesson 18)
- EQ_{DFAREX} (Exercise 4.2) (or any combination therein)
- ALL_{DFA} (Exercise 4.3)
- $A\epsilon_{\text{CFG}}$ (Exercise 4.4)
- $\{\langle R, S\rangle \mid R, S \text{ are regex} \wedge L(R) \subseteq L(S)\}$ is decidable (Problem 4.13)
- $\{\langle R\rangle \mid R \text{ is regex over } \{0,1\} \wedge w \text{ contains } 111 \wedge w \in L(G)\}$ (Exercise 4.16)

Exercise 3 (Level 1)

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Give an example of a recognizable and undecidable language.

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Give an example of a recognizable and undecidable language.

Solution: A_{TM} is recognizable (in proof of Theorem 4.11, page 202) and undecidable (Theorem 4.11).

Tip: build a table of (co-)recognizable, decidable, undecidable, and (co-)unrecognizable languages

- Think of A, E, EQ for DFA, CFG, and TM

Exercise 4 (Level 2)

> Map-reducible: Use decidability (Theorem 5.22 and Corollary 5.23) and recognizability (Theorem 5.28 and Corollary 5.29) to derive conclusions about the languages we studied ($A, E, EQ + DFA, CFG, TM$).

Exercise 4 (Level 2)

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Given that $A_{\text{TM}} \leq_{\mathrm{m}} HALT_{\text{TM}}$, show that $HALT_{\text{TM}}$ is undecidable.

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> Map-reducible: Use decidability (Theorem 5.22 and Corollary 5.23) and recognizability (Theorem 5.28 and Corollary 5.29) to derive conclusions about the languages we studied ($A, E, EQ + DFA, CFG, TM$).

Given that $A_{\mathsf{TM}} \leq_{\mathrm{m}} HALT_{\mathsf{TM}}$, show that $HALT_{\mathsf{TM}}$ is undecidable.

Proof. Apply Corollary 5.23 since A_{TM} is undecidable (Theorem 4.11) and $A_{\mathsf{TM}} \leq_{\mathrm{m}} HALT_{\mathsf{TM}}$ (hypothesis).

More examples

- Show that $\overline{HALT}_{\mathsf{TM}}$ is unrecognizable.
- Show that $HALT_{\mathsf{TM}}$ is undecidable. (Exercise 5.24/Lesson 22)
- Show that A_{TM} is recognizable via mapping reducibility. (Lesson 22)

Exercise 5 (level 2)

- Relate facts on map-reducible.

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- Exercise 5.6: $\leq_\mathrm{m}$ is a transitive relation.
- Exercise 5.22: A is recognizable iff $A \leq_\mathrm{m} A_\mathsf{TM}$.

Let (H1) $A_{CFG} \leq_\mathrm{m} A_{TM}$, (H2) $A_{DFA} \leq_\mathrm{m} A_{CFG}$, and (H3) A_{TM} is recognizable. Prove that we can conclude that A_DFA is recognizable using map-reducibility.

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- Exercise 5.22: A is recognizable iff $A \leq_{\text{m}} A_{\text{TM}}$.

Let (H1) $A_{CFG} \leq_{\text{m}} A_{TM}$, (H2) $A_{DFA} \leq_{\text{m}} A_{CFG}$, and (H3) A_{TM} is recognizable.
Prove that we can conclude that A_{DFA} is recognizable using map-reducibility.

Proof.

1. $A_{DFA} \leq_{\text{m}} A_{TM}$ by Exercise 5.6, (H1) $A_{CFG} \leq_{\text{m}} A_{TM}$, (H2) $A_{DFA} \leq_{\text{m}} A_{CFG}$.
2. A_{DFA} is recognizable, by Exercise 5.22, (1) $A_{DFA} \leq_{\text{m}} A_{TM}$, and (H3).

Exercise 6 (Level 2)

- Relate facts on map-reducible.

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Relate facts on map-reducible.

- Lemma R.1: If $A \leq_{\mathrm{m}} B$, then $\overline{A} \leq_{\mathrm{m}} \overline{B}$.
- Lemma R.2: If $A \leq_{\mathrm{m}} \overline{B}$ and B recognizable, then $\overline{A} \leq_{\mathrm{m}} B$.
- Lemma R.3: If A recognizable and $\overline{A} \leq_{\mathrm{m}} B$, then $A \leq_{\mathrm{m}} \overline{B}$.

Let (H1) $B \leq \overline{A}_{TM}$. Show that $\overline{B}$ is recognizable.

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Let (H1) $B \leq \overline{A}_{TM}$. Show that $\overline{B}$ is recognizable.

Proof.

1. $\overline{B} \leq A_{TM}$, by Lemma R.2, (H1) $\overline{A}_{TM} \leq B$, and A_{TM} recognizable (pp 202).
2. $\overline{B}$ is recognizable, by Exercise 5.22 and (1) $\overline{B} \leq A_{TM}$.

Exercise 7 (Level 2)

- Relate facts on map-reducible.

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Relate facts on map-reducible.

Show that $\overline{HALT}_{\mathsf{TM}}$ is unrecognizable.

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Relate facts on map-reducible.

Show that $\overline{HALT}_{\text{TM}}$ is unrecognizable.

Proof.

1. $\overline{A}_{\text{TM}} \leq_{\text{m}} \overline{HALT}_{\text{TM}}$, by Theorem R.1 and $A_{\text{TM}} \leq_{\text{m}} HALT_{\text{TM}}$ (exercise 5.24)
2. $\overline{HALT}_{\text{TM}}$ is unrecognizable, by Corollary 5.29, $\overline{A}_{\text{TM}} \leq_{\text{m}} \overline{HALT}_{\text{TM}}$ (1), and $\overline{A}_{\text{TM}}$ is unrecognizable (Corollary 4.23)

Exercise 8 (Level 3)

(Exercise 4.2 in the book.)

$$EQ_{DFAREX}\{\langle D, R\rangle \mid D \text{ is a DFA} \wedge R \text{ is a regex} \wedge L(D) = L(R)\}$$

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(Exercise 4.2 in the book.)

$$EQ_{DFAREX}\{\langle D, R\rangle \mid D \text{ is a DFA} \land R \text{ is a regex} \land L(D) = L(R)\}$$

Let $r2n$ be the function that converts a regular expression into an NFA and $n2d$ be the function that converts an NFA into a DFA.

1. $EQ_{DFAREX} \leq_m EQ_{DFA}$ with $F(\langle D, R\rangle) = \langle D, n2d(r2n(R))\rangle$.
   - Unfold $\leq_m$. Goal: $\langle D, R\rangle \in EQ_{DFAREX} \iff F(\langle D, R\rangle) \in EQ_{DFA}$
   - Unfold EQ_{DFAREX}, EQ_{DFA}, and F. Goal: $L(D) = L(R) \iff L(D) = n2d(r2n(R))$
   - Rewrite goal with $\forall N, L(n2d(N)) = L(N)$ and $\forall R, r2n(R) = L(R)$. Goal: $L(D) = L(R) \iff L(D) = L(R)$. Proof: trivial, since $\forall P, P \iff P$.
2. EQ_{DFAREX} is decidable, by Theorem 5.22, (1) $EQ_{DFAREX} \leq_m EQ_{DFA}$, and EQ_{DFA} decidable (Theorem 4.5).

> The proof has two main parts: 1) showing that the given language map-reduces to a decidable language and 2) use Theorem 5.22 to conclude.

Exercise 8 (Level 3)

Continuation...

- The proof has two main parts: 1) showing that the given language map-reduces to a decidable language and 2) use Theorem 5.22 to conclude.
- Whenever you say that $A \leq_m B$ be clear about which **function** reduces A to B.

More examples

- See HW7

Exercise 9 (Level 3)

Hint...

> Combine Lemma R.1, R.2, R.3, Exercise 5.6, Exercise 5.22, and decidability, recognizability to relate the recognizability/decidability between mapping-reducible languages.

Thank you!