CS420

Introduction to the Theory of Computation

Lecture 4: Nondeterministic Finite Automaton

Tiago Cogumbreiro

Revisiting what we learned...

- Operations on words; set theory
- How to draw a state diagram from a DFA?
- A step-by-step union example
- Reduction graphs with ϵ-transitions?
- What is the powerset function?
- How to draw a state diagram from an NFA?
- How to convert an NFA into a DFA?

HW1 heads up

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- Answers should all be given as state diagrams
- Simplification of L_5:
  - If the resulting DFA is already simplified, then just answer "the same DFA"
  - We have no way of proving that the DFA is the smallest
- After applying the union operator you should **not** simplify the final diagram
- When writing down a diagram from an M (directly using a transition function) you should **not** simplify it

Operations on words; set theory

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What is w^n; $L^\star$

Operations on words; set theory

What is w^n; $L^\star$

Answer: (Lecture 1, slides 38 and 39) concatenate w with itself n times. $L^\star = \{w^n \mid w \in L \wedge n \geq 0\}$ (Lecture 2; slide 19)

See also **Definition 1.23** in the book.

Operations on words; set theory

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What is $w_1, w, w_2 \in s$?

Operations on words; set theory

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What is $w_1, w, w_2 \in s$?

Answer: Shorthand notation for $w_1 \in s \wedge w \in s \wedge w_2 \in s$.

Operations on words; set theory

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What is $w_1, w, w_2 \in s$?

Answer: Shorthand notation for $w_1 \in s \land w \in s \land w_2 \in s$.

What is $\{w \mid P(w)\}$?

Answer: this is known as the **set-builder notation** (set comprehension). I assume you learned this in CS220 (or prior). It is a way of saying any w such that $P(x)$ holds. For instance, $\{w \mid |w| \text{ is even} \land w = ab \cdot w_2 \land w, w_2 \in \Sigma^\star\}$ means that w is such that: $|w|$ is even (the length of w is even) **AND** $w = ab \cdot w_2$ (w starts with ab followed by w_2), **AND** $w_2 \in \Sigma^\star$ (w_2 is a word)

How to draw a state diagram for a DFA

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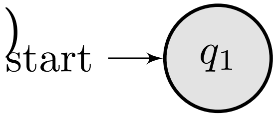
Give the DFA of $M = (\{q_1, q_2, q_3, q_4, q_5\}, \{\mathtt{a}, \mathtt{b}\}, \delta, q_1, \{q_4\})$ where



$$
\begin{aligned}
\delta(q_1, \mathtt{a}) &= q_2 \\
\delta(q_1, \mathtt{b}) &= q_3 \\
\delta(q_2, \mathtt{a}) &= q_1 \\
\delta(q_2, \mathtt{b}) &= q_3 \\
\delta(q_3, \mathtt{a}) &= q_5 \\
\delta(q_4, \mathtt{a}) &= q_1 \\
\delta(q_4, \mathtt{b}) &= q_2 \\
\delta(q, c) &= q \text{ otherwise}
\end{aligned}
$$

1. pick q_1; draw edge for each Σ, one for a; another for b

How to draw a state diagram for a DFA

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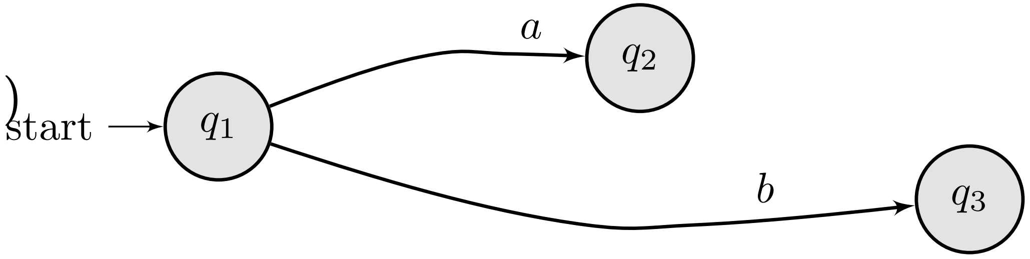
\delta(q_4, \mathtt{b}) &= q_2 \\
\delta(q, c) &= q \text{ otherwise}
\end{aligned}
$$



1. pick q_1; draw outgoing edges
2. pick q_2 (or q_3); draw outgoing edges

How to draw a state diagram for a DFA

Give the DFA of $M = (\{q_1, q_2, q_3, q_4, q_5\}, \{\mathtt{a}, \mathtt{b}\}, \delta, q_1, \{q_4\})$ where

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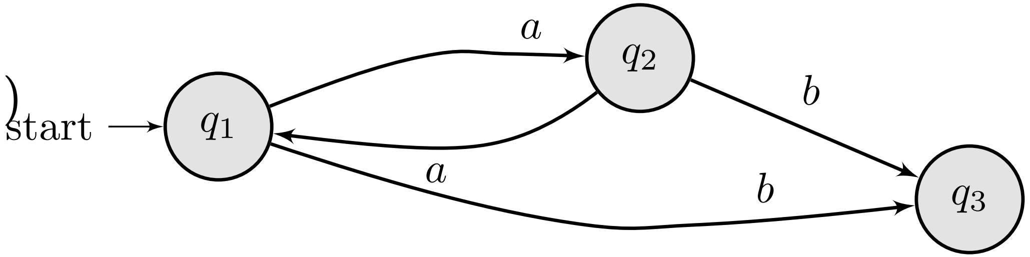
$$



1. pick q_1; draw outgoing edges
2. pick q_2; draw outgoing edges
3. pick q_3; draw outgoing edges

How to draw a state diagram for a DFA

Give the DFA of $M = (\{q_1, q_2, q_3, q_4, q_5\}, \{\mathtt{a}, \mathtt{b}\}, \delta, q_1, \{q_4\})$ where

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\delta(q_3, \mathtt{a}) &= q_5 \\
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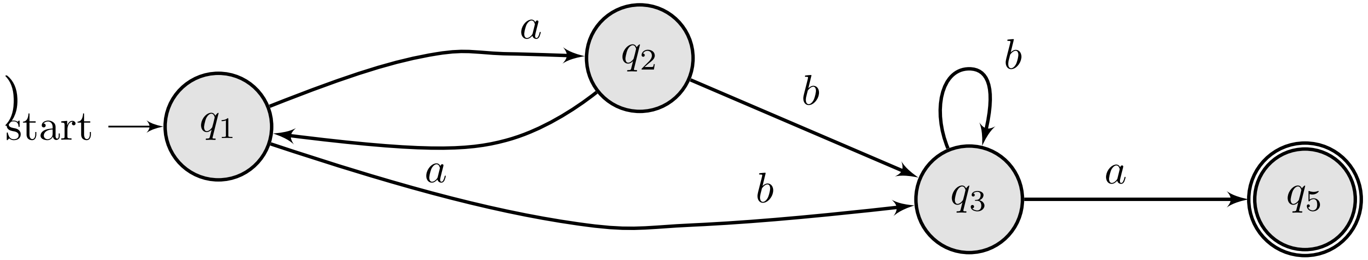
\delta(q_4, \mathtt{b}) &= q_2
\end{aligned}
$$

$$\delta(q, c) = q \text{ otherwise}$$



1. pick q_1; draw outgoing edges
2. pick q_2; draw outgoing edges
3. pick q_3; draw outgoing edges
4. pick q_5; draw outgoing edges

How to draw a state diagram for a DFA

Give the DFA of $M = (\{q_1, q_2, q_3, q_4, q_5\}, \{\mathtt{a}, \mathtt{b}\}, \delta, q_1, \{q_4\})$ where

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\begin{aligned}
\delta(q_1, \mathtt{a}) &= q_2 \\
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\delta(q_2, \mathtt{b}) &= q_3 \\
\delta(q_3, \mathtt{a}) &= q_5 \\
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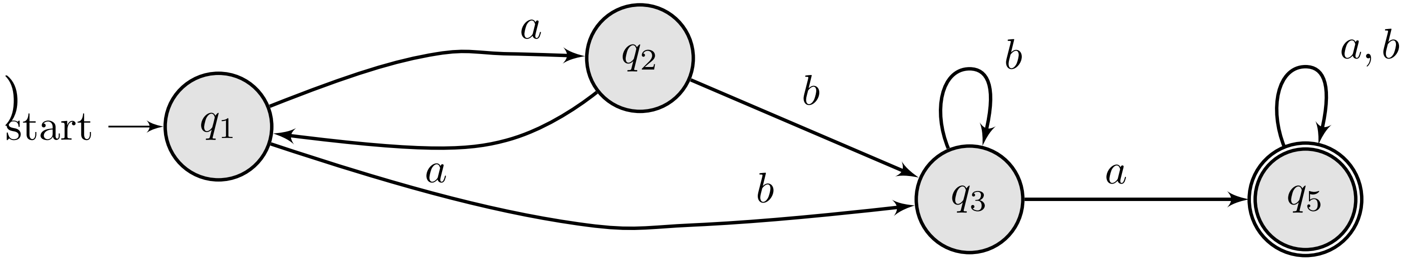
\delta(q_4, \mathtt{b}) &= q_2 \\
\delta(q, c) &= q \text{ otherwise}
\end{aligned}
$$



1. pick q_1; draw outgoing edges
2. pick q_2; draw outgoing edges
3. pick q_3; draw outgoing edges
4. pick q_5; draw outgoing edges

Note 1: state q_4 is not present in our graph, because it is unreachable. We only render reachable states in our state diagrams.
Note 2: do **not** attempt to simplify the DFA.

Step-by-step union example

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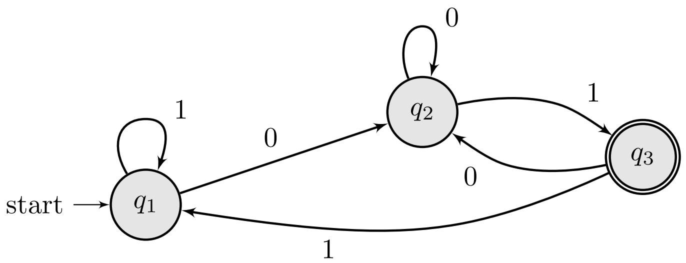
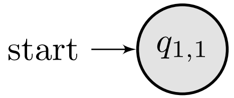
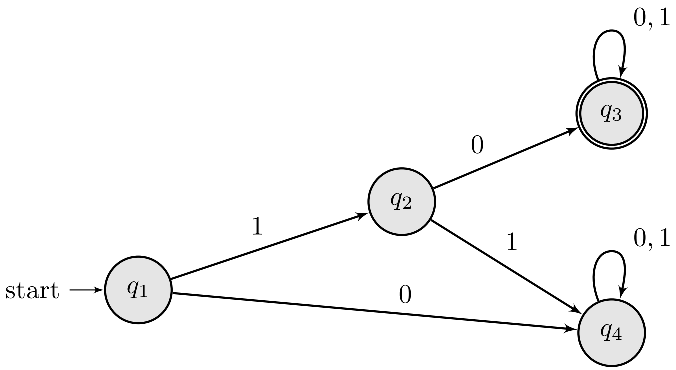
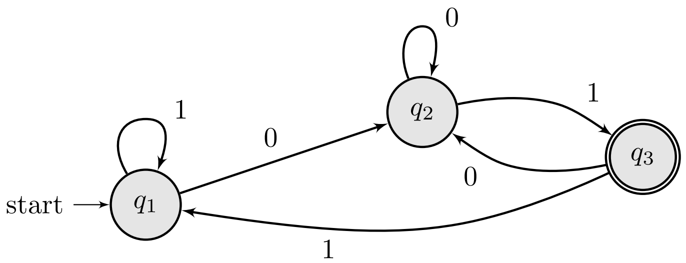


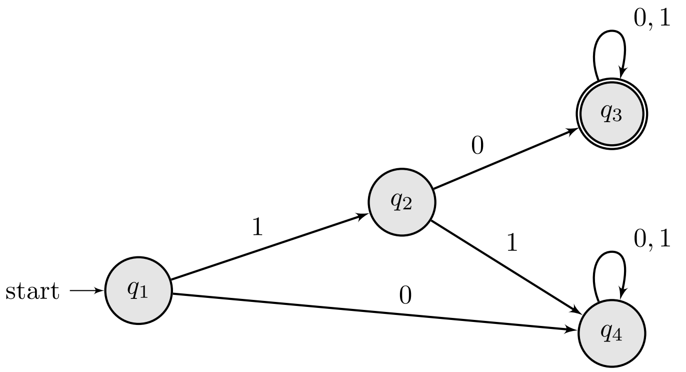


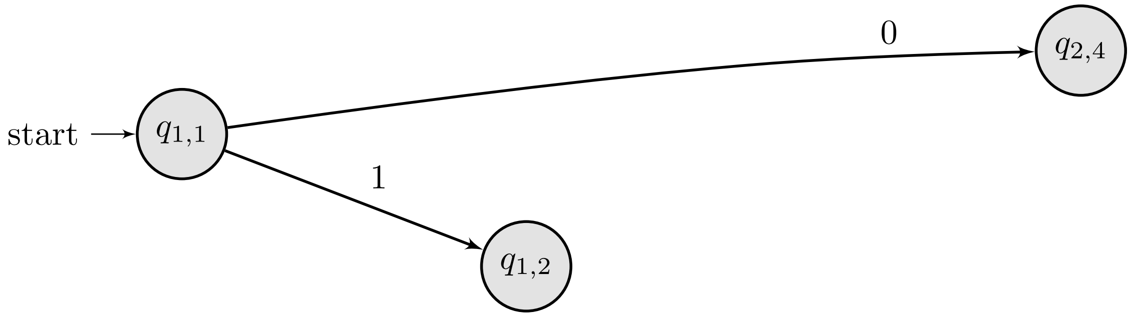


> We start from the pair (q_1, q_1) (the initial state of each DFA) which we denote by $q_{1,1}$.
> For each element of Σ draw an edge.

Step-by-step union example



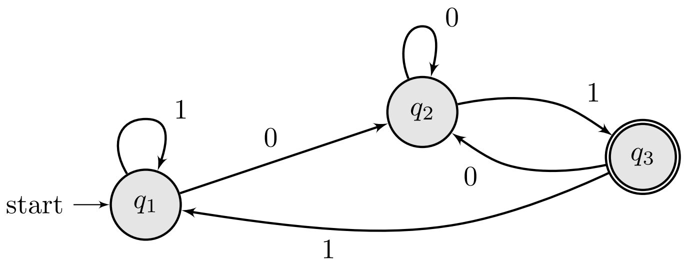


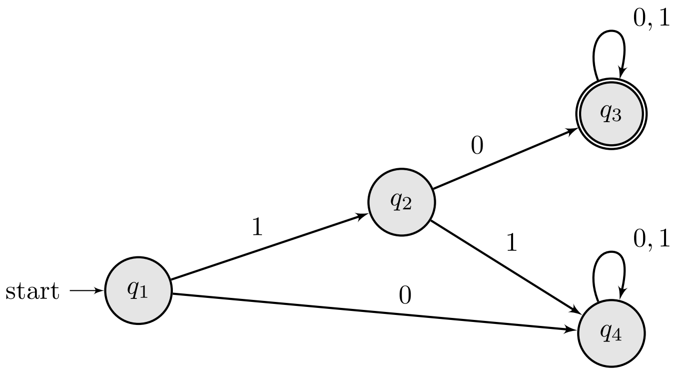


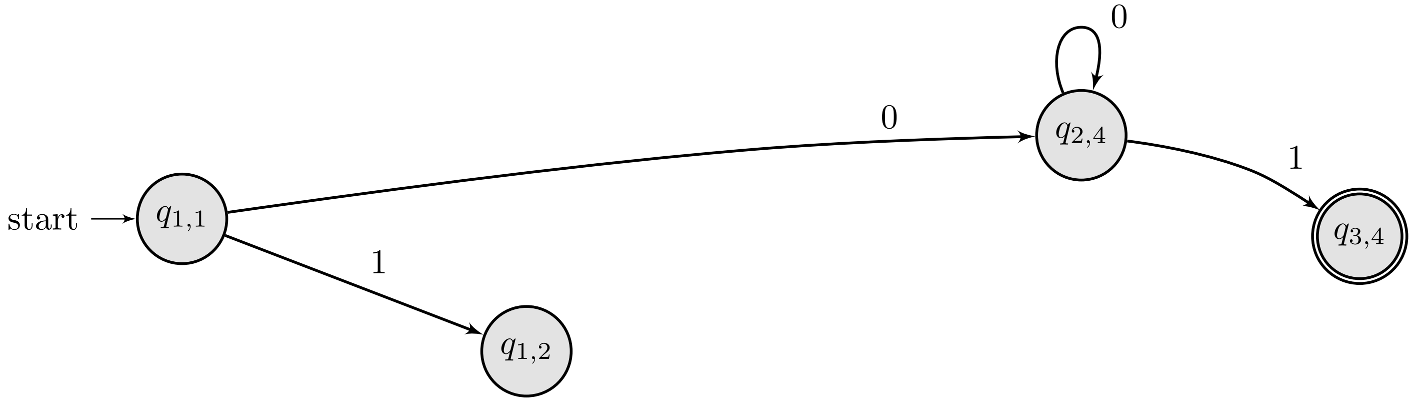
At $q_{1,1}$

- Read `0`. (Left) From q_1 we advance to q_2. (Right) From q_1 we advance to q_4. Result $q_{2,4}$
- Read 1. (Left) From q_1 we advance to q_1. (Right) From q_1 we advance to q_2. Result $q_{1,2}$.

Step-by-step union example



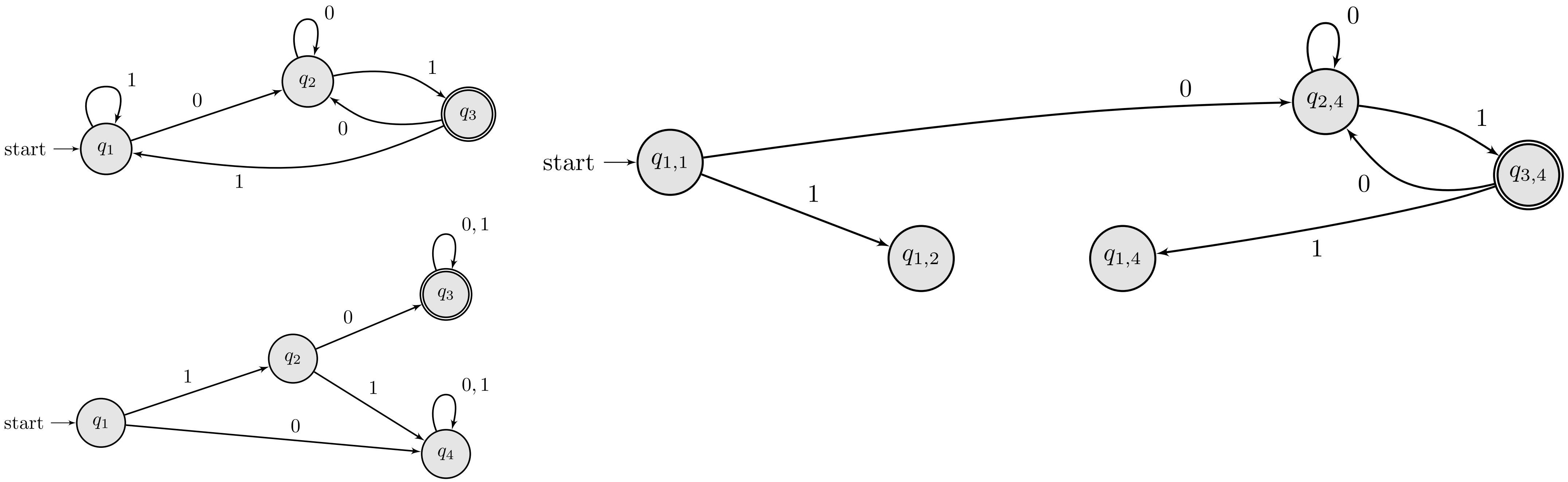




At $q_{2,4}$:

- Read `0`. (Left) From q_2 we advance to q_2. (Right) From q_4 we advance to q_4. Result $q_{2,4}$
- Read `1`. (Left) From q_2 we advance to q_3. (Right) From q_4 we advance to q_4. Result $q_{3,4}$.

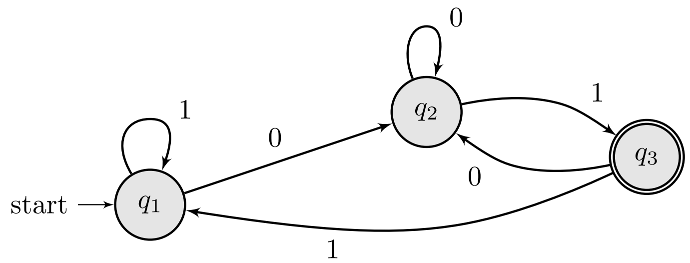
Step-by-step union example

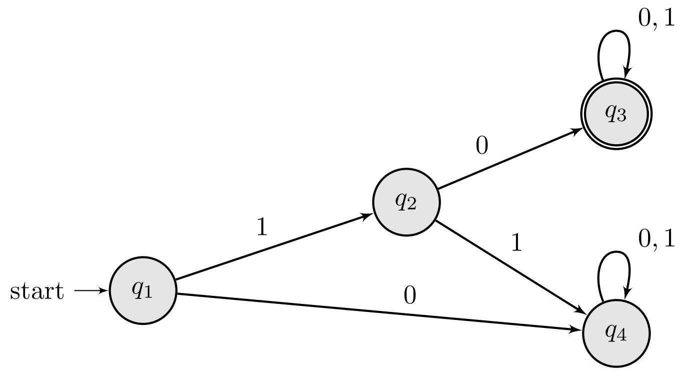


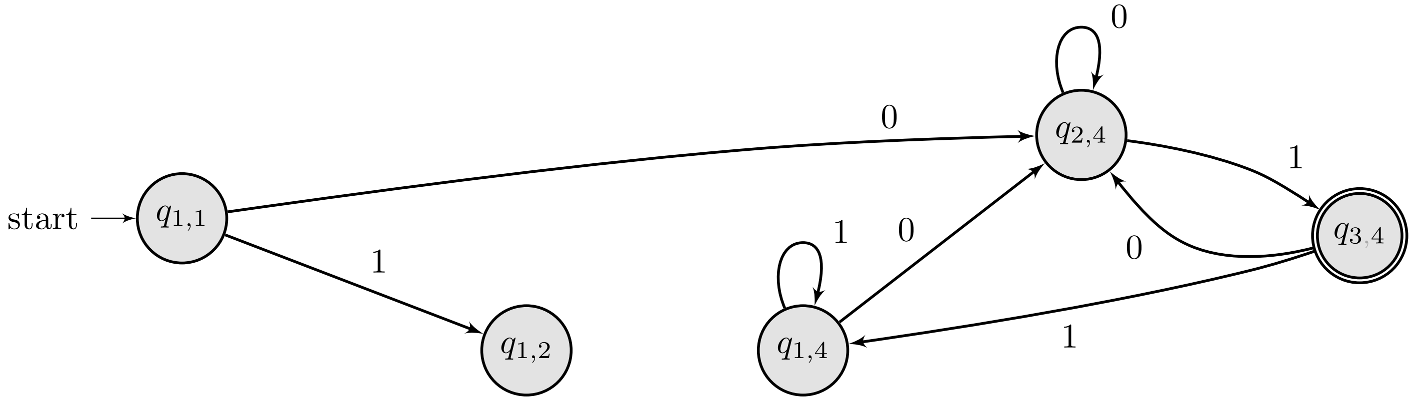
At $q_{3,4}$:

- Read `0`. (Left) From q_3 we advance to q_2. (Right) From q_4 we advance to q_4. Result $q_{2,4}$
- Read 1. (Left) From q_3 we advance to q_1. (Right) From q_4 we advance to q_4. Result $q_{1,4}$.

Step-by-step union example







At $q_{1,4}$:

- Read `0`. (Left) From q_1 we advance to q_2. (Right) From q_4 we advance to q_4. Result $q_{2,4}$
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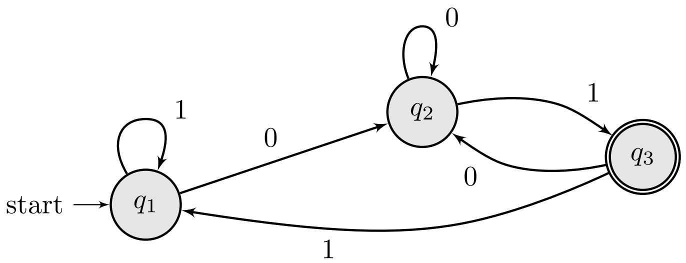
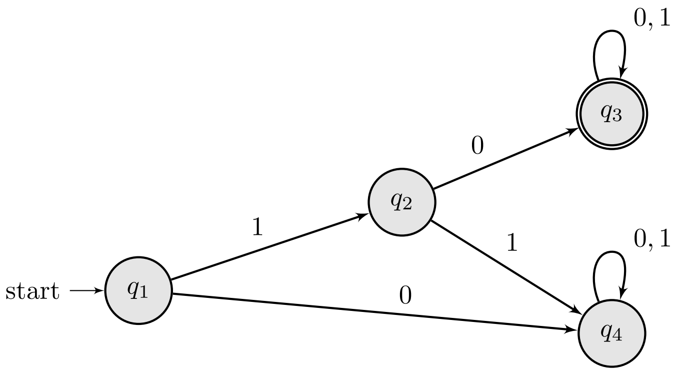
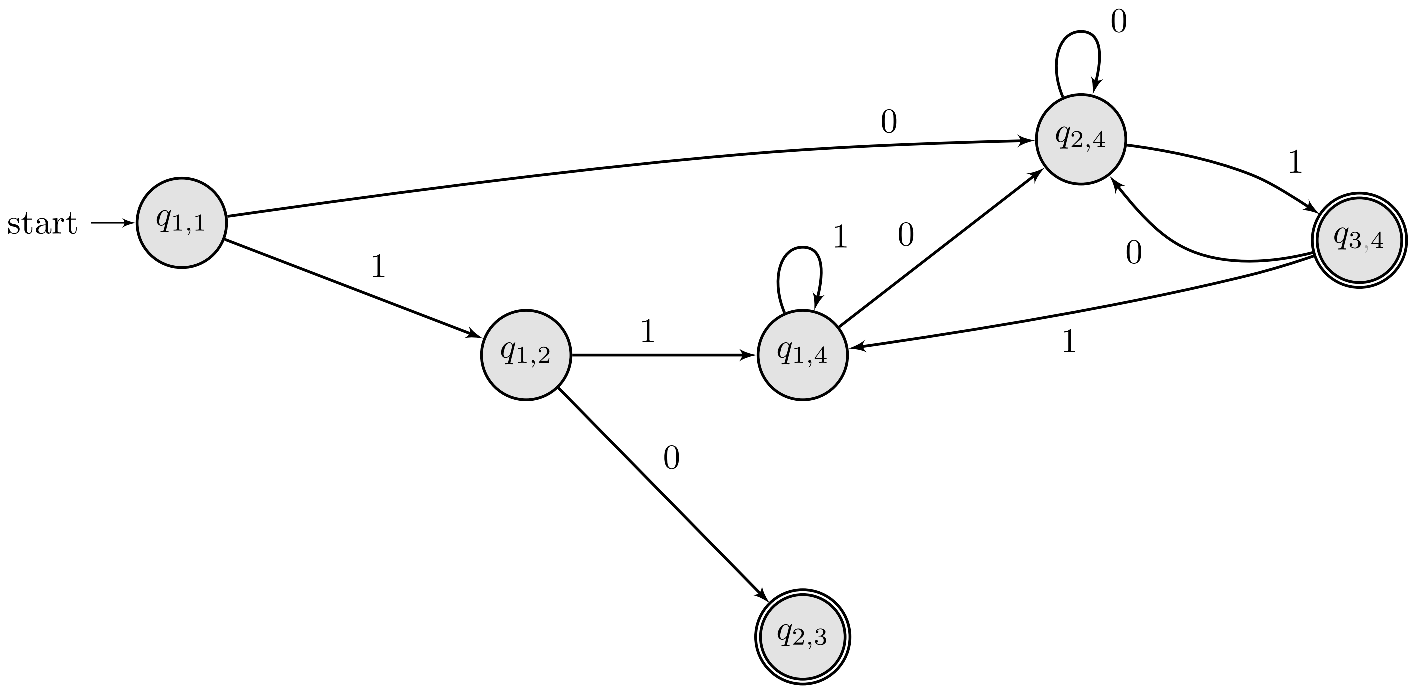
Step-by-step union example



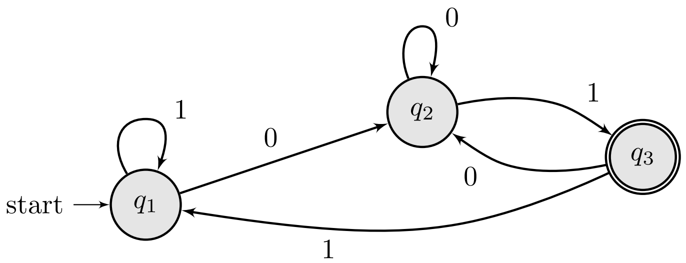


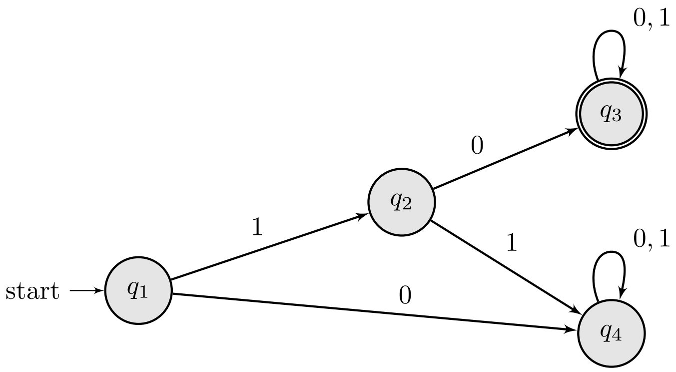


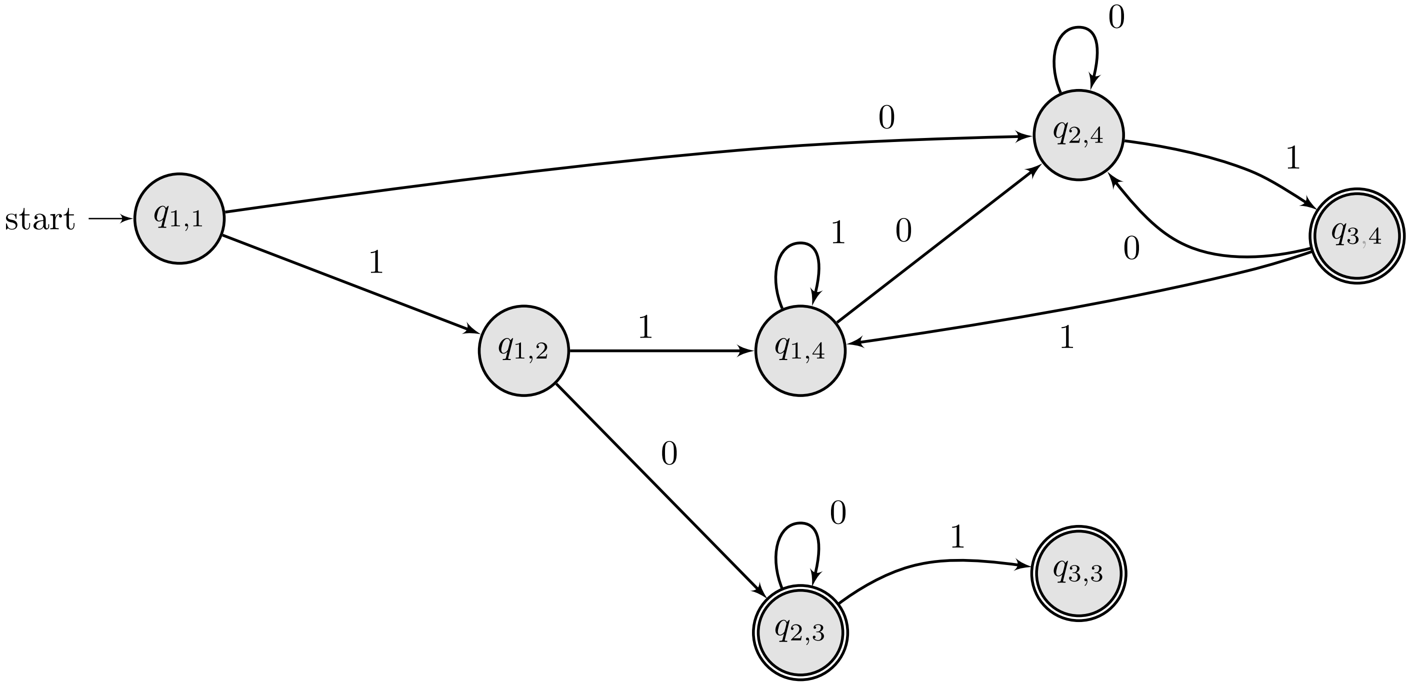
At $q_{1,2}$:

- Read 0. (Left) From q_1 we advance to q_2. (Right) From q_2 we advance to q_3. Result $q_{2,3}$
- Read 1. (Left) From q_1 we advance to q_1. (Right) From q_2 we advance to q_4. Result $q_{1,4}$.

Step-by-step union example

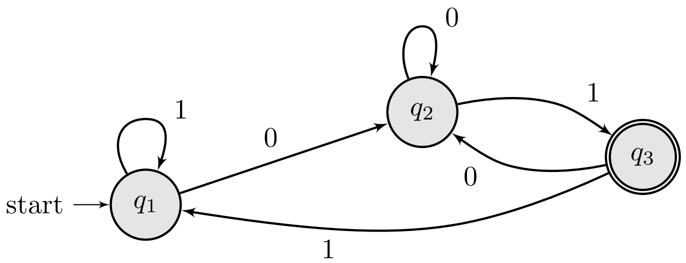


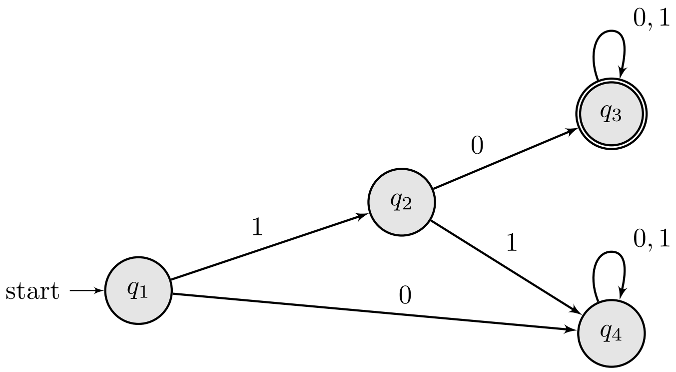


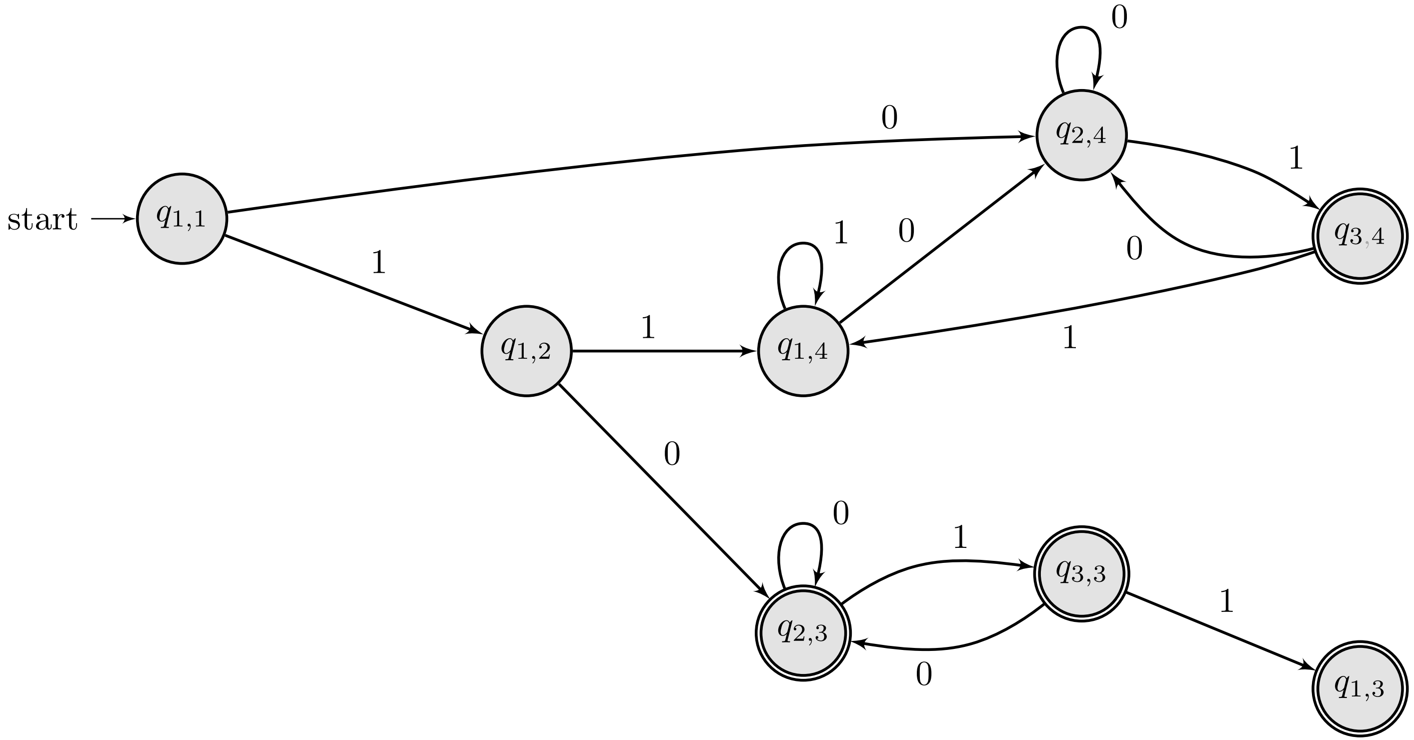


- Read `0`. (Left) From q_2 we advance to q_2. (Right) From q_3 we advance to q_3. Result $q_{2,3}$
- Read 1. (Left) From q_2 we advance to q_3. (Right) From q_3 we advance to q_3. Result $q_{3,3}$.

Step-by-step union example

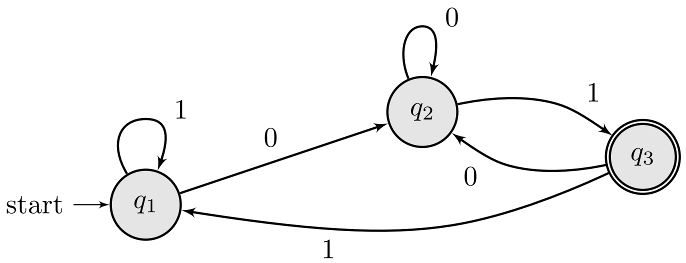


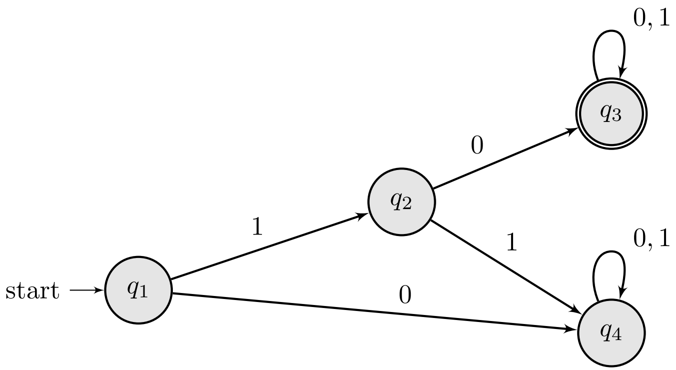


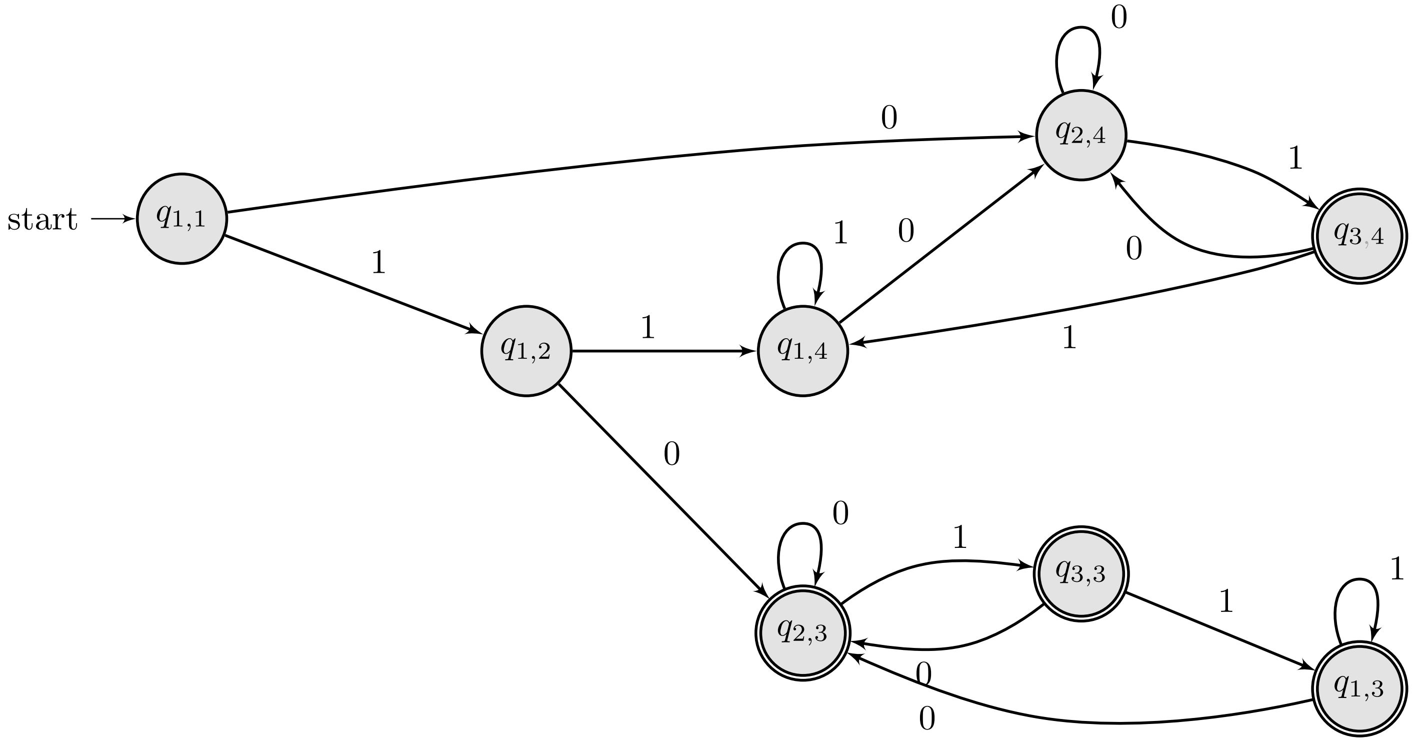


- Read `0`. (Left) From q_3 we advance to q_2. (Right) From q_3 we advance to q_3. Result $q_{2,3}$
- Read 1. (Left) From q_3 we advance to q_1. (Right) From q_3 we advance to q_3. Result $q_{1,3}$.

Step-by-step union example







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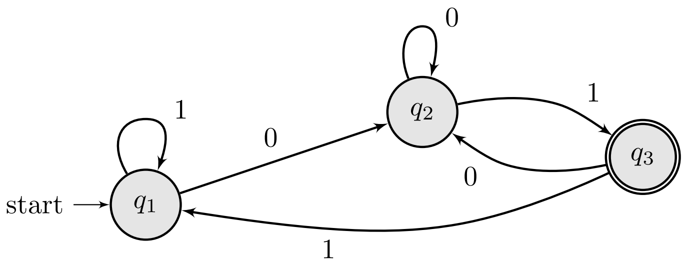
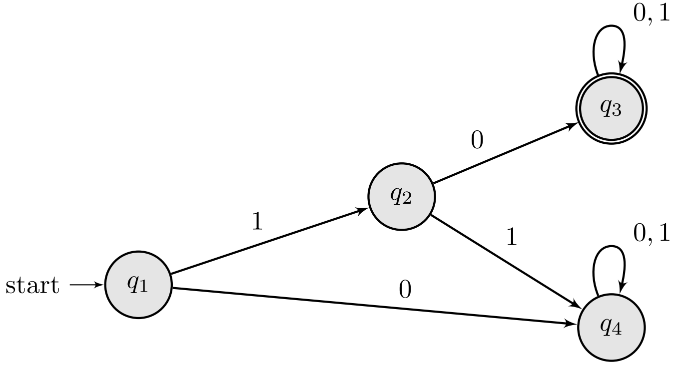
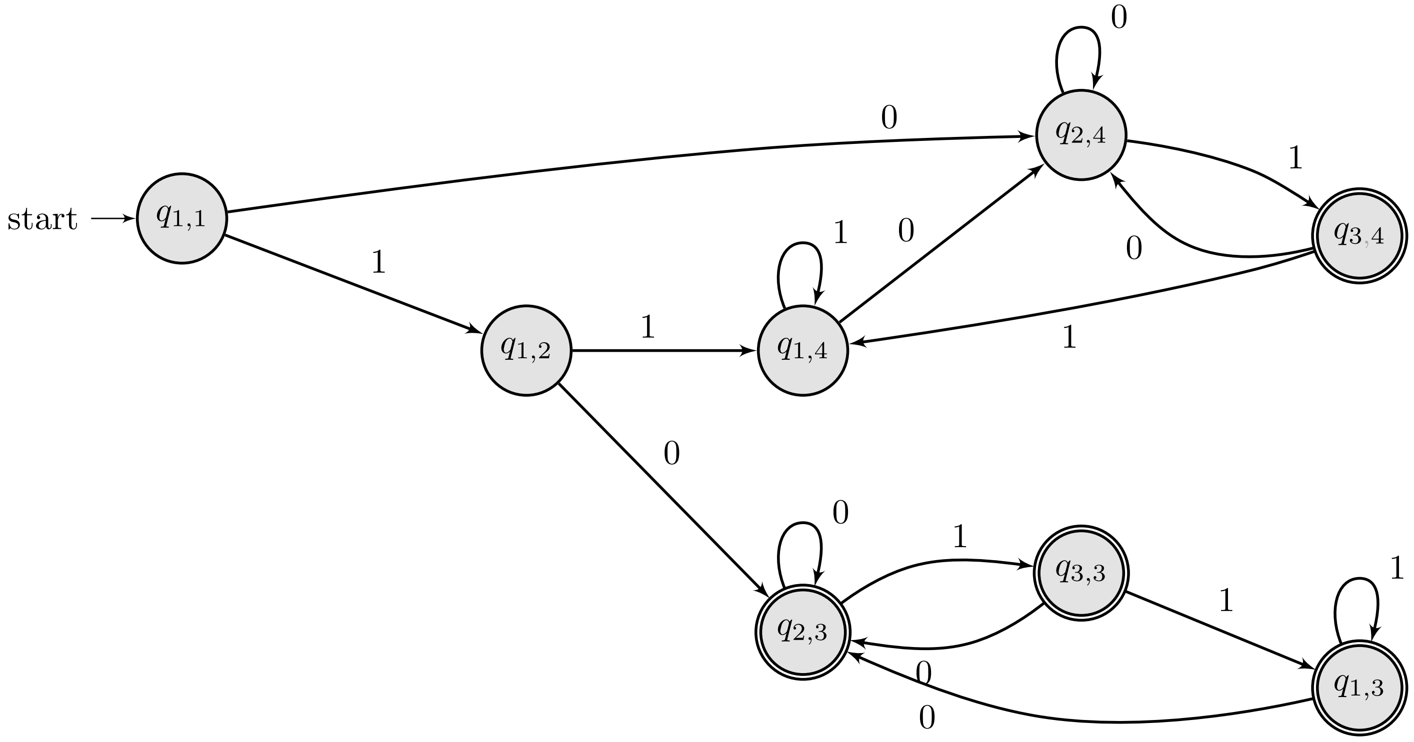
Step-by-step union example



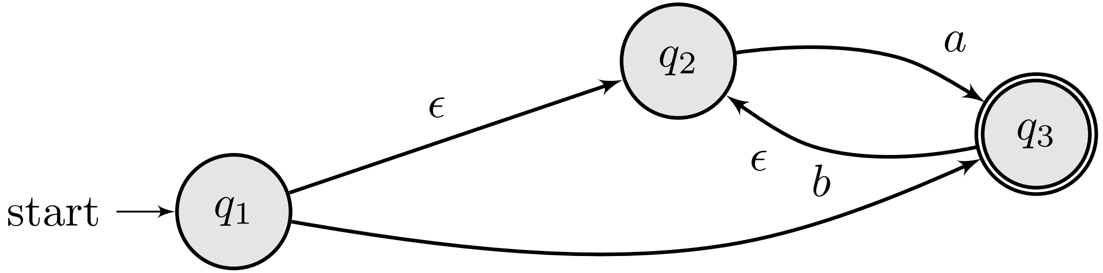




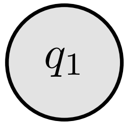
Note: in the HW/mini-tests do **not** attempt to simplify the resulting DFA unless explicitly requested to do so.

Reduction graphs with ϵ-transitions

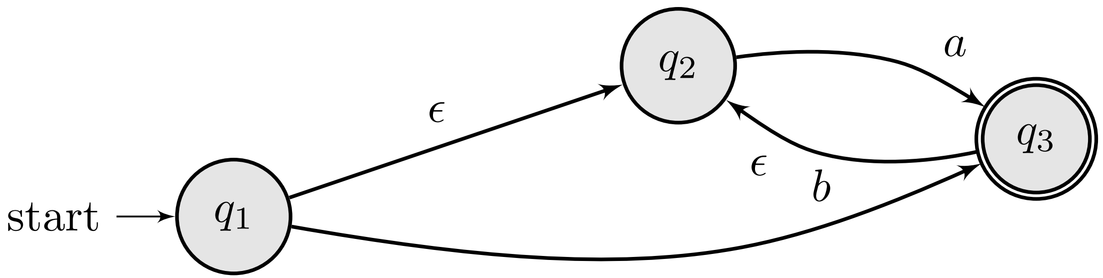
Reduction graphs with ϵ-transitions



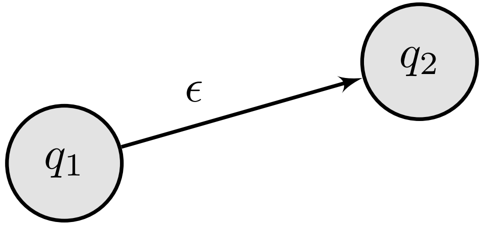
Acceptance for ba: epsilon-step



Reduction graphs with ϵ-transitions

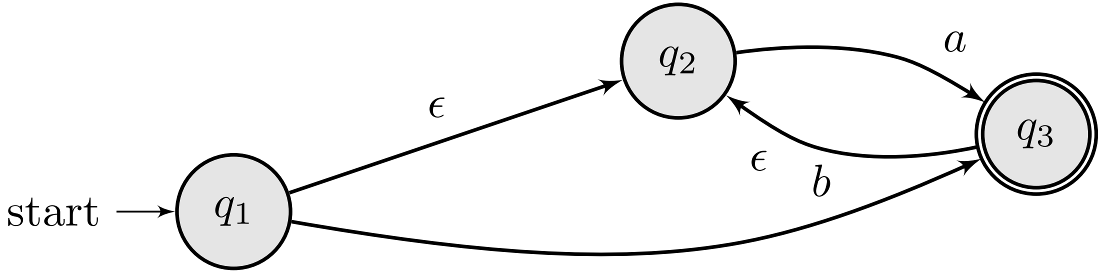


Acceptance for ba: input-step **b**

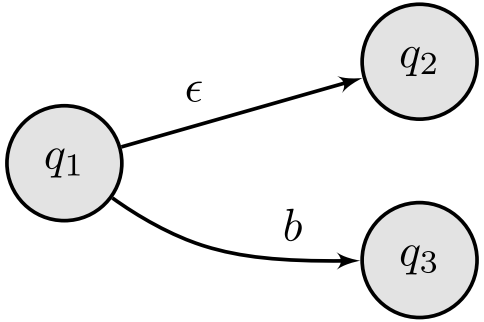


> **Note:** at this point that are two concurrent states: q_1 and q_2, so we can consume b from either (although we can only do so via q_1).

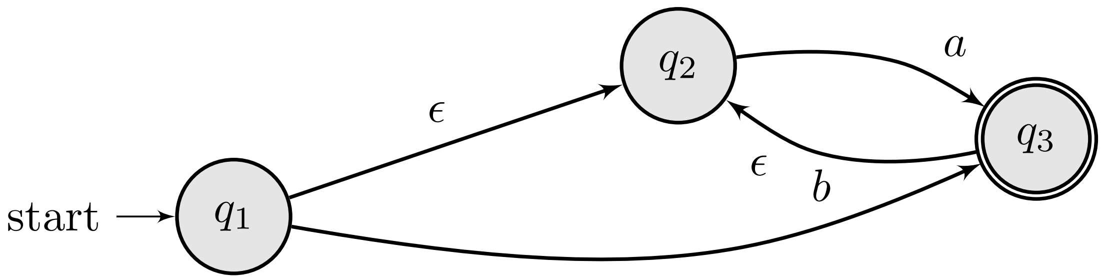
Reduction graphs with ϵ-transitions



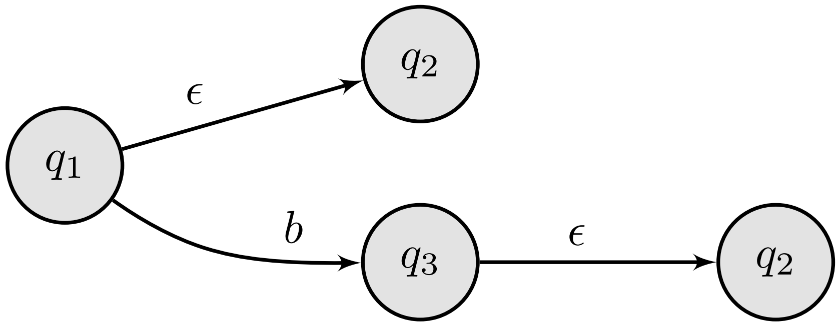
Acceptance for ba: epsilon-step



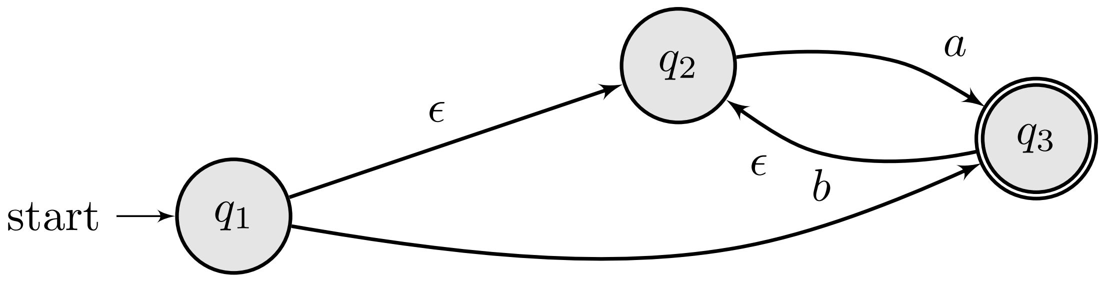
Reduction graphs with ϵ-transitions



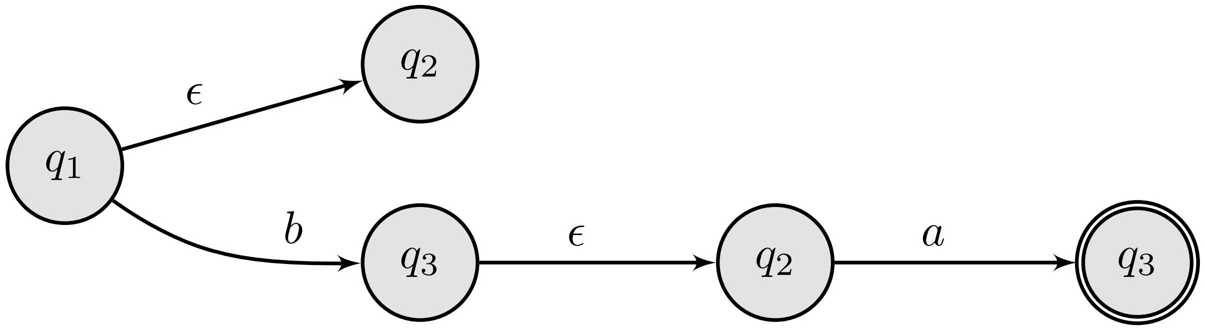
Acceptance for ba: input-step



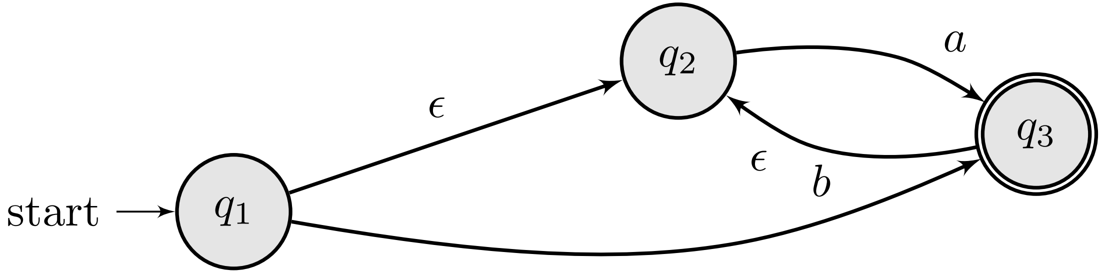
Reduction graphs with ϵ-transitions



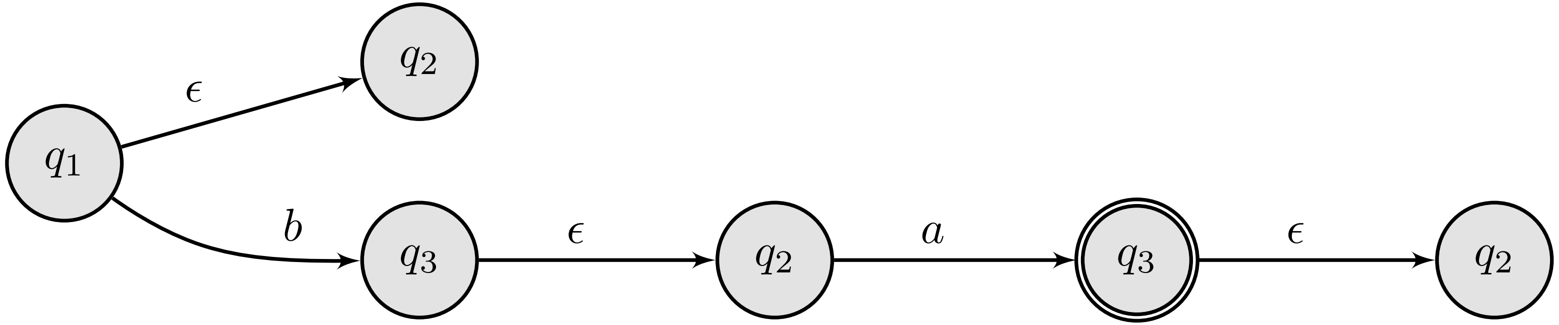
Acceptance for ba: epsilon-step



Reduction graphs with ϵ-transitions



Acceptance for ba



What is the Powerset function?

What is the Powerset function?

Given a set it returns a set that consists of all possible subsets of that set and itself.

$$\mathcal{P}(s) = \{r \mid r \subseteq s\}$$

Example

$\mathcal{P}(\{q_1, q_2, q_3\}) =$

What is the Powerset function?

Given a set it returns a set that consists of all possible subsets of that set and itself.

$$\mathcal{P}(s) = \{r \mid r \subseteq s\}$$

Example

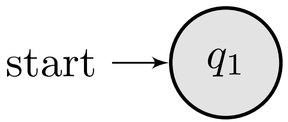
$$\mathcal{P}(\{q_1, q_2, q_3\}) = \{\emptyset, \{q_1\}, \{q_2\}, \{q_3\}, \{q_1, q_2\}, \{q_1, q_3\}, \{q_2, q_3\}, \{q_1, q_2, q_3\}\}$$

How to draw
a state diagram for an NFA

How to draw a state diagram for an NFA

$M = (\{q_1, q_2, q_3, q_4\}, \{\mathtt{a}, \mathtt{b}\}, \delta, q_1, \{q_2, q_4\})$ where

$$\delta(q_1, \mathtt{a}) = \{q_1, q_2, q_3\}$$
$$\delta(q_1, \epsilon) = \{q_3\}$$
$$\delta(q_2, \mathtt{a}) = \{q_1, q_3\}$$
$$\delta(q_2, \mathtt{b}) = \{q_2\}$$
$$\delta(q_3, \mathtt{b}) = \{q_4\}$$
$$\delta(q_3, \epsilon) = \{q_1\}$$
$$\delta(q, c) = \emptyset \text{ otherwise}$$



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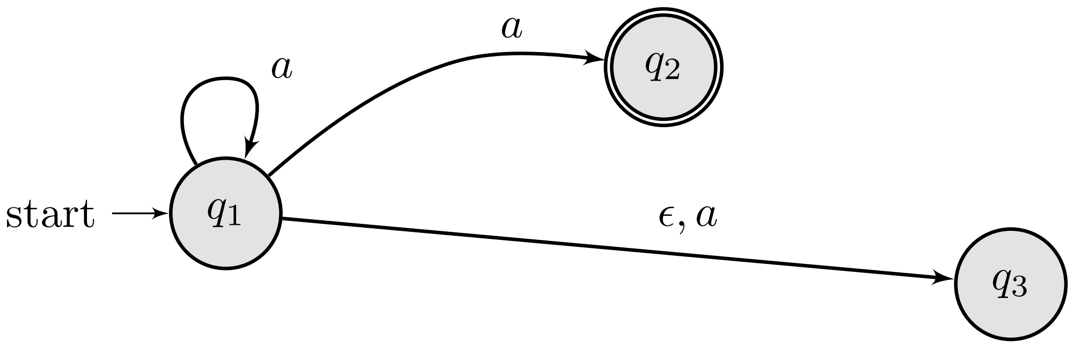

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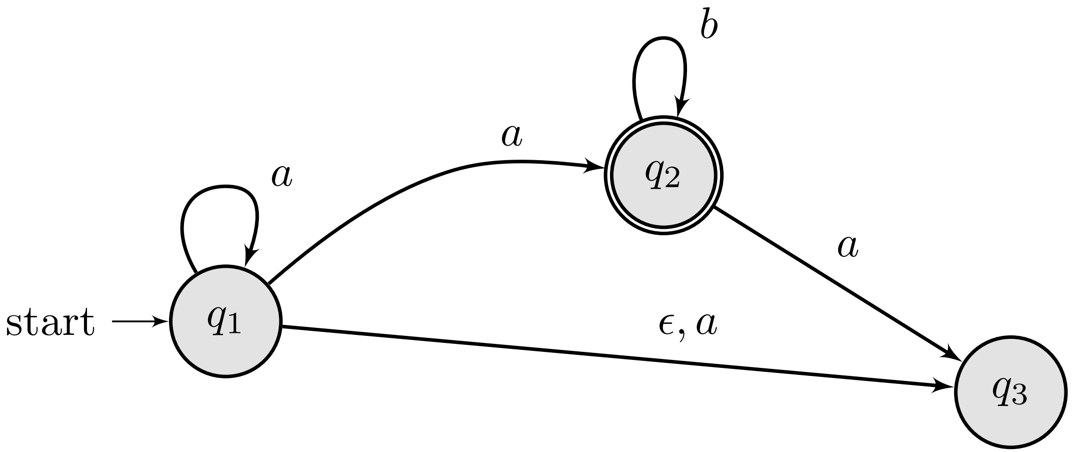

How to draw a state diagram for an NFA

$M = (\{q_1, q_2, q_3, q_4\}, \{\mathtt{a}, \mathtt{b}\}, \delta, q_1, \{q_2, q_4\})$ where

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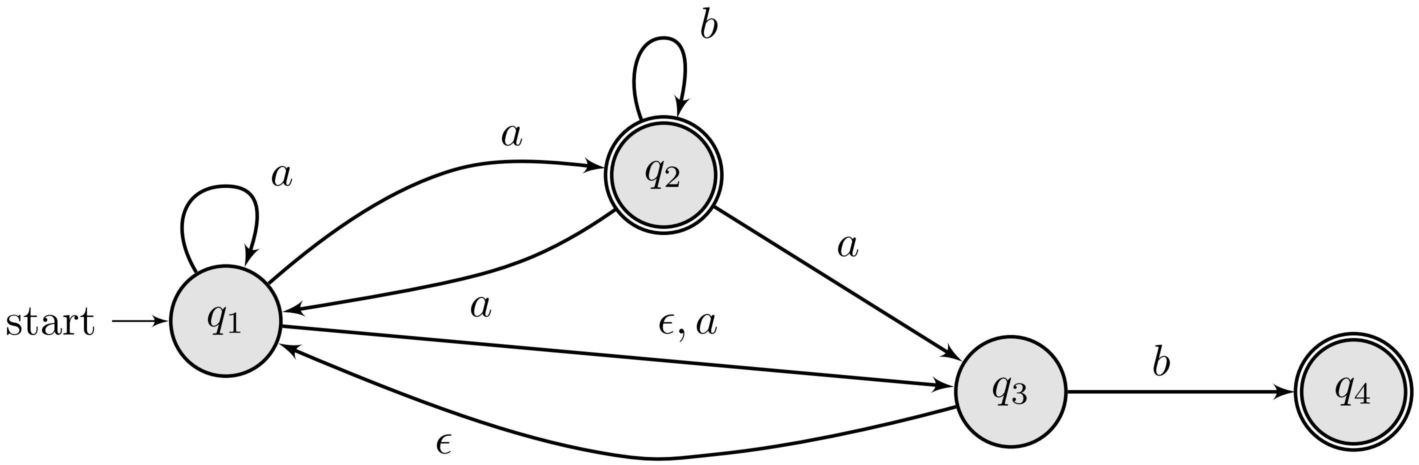
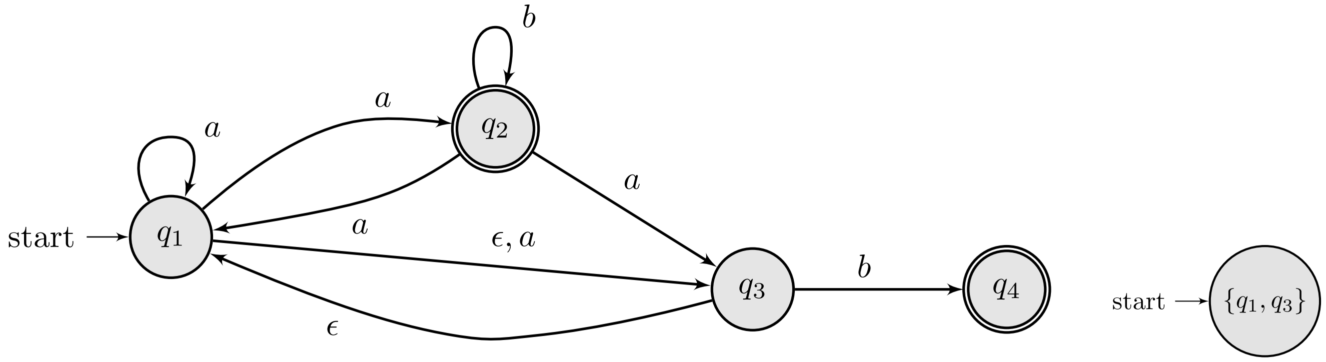


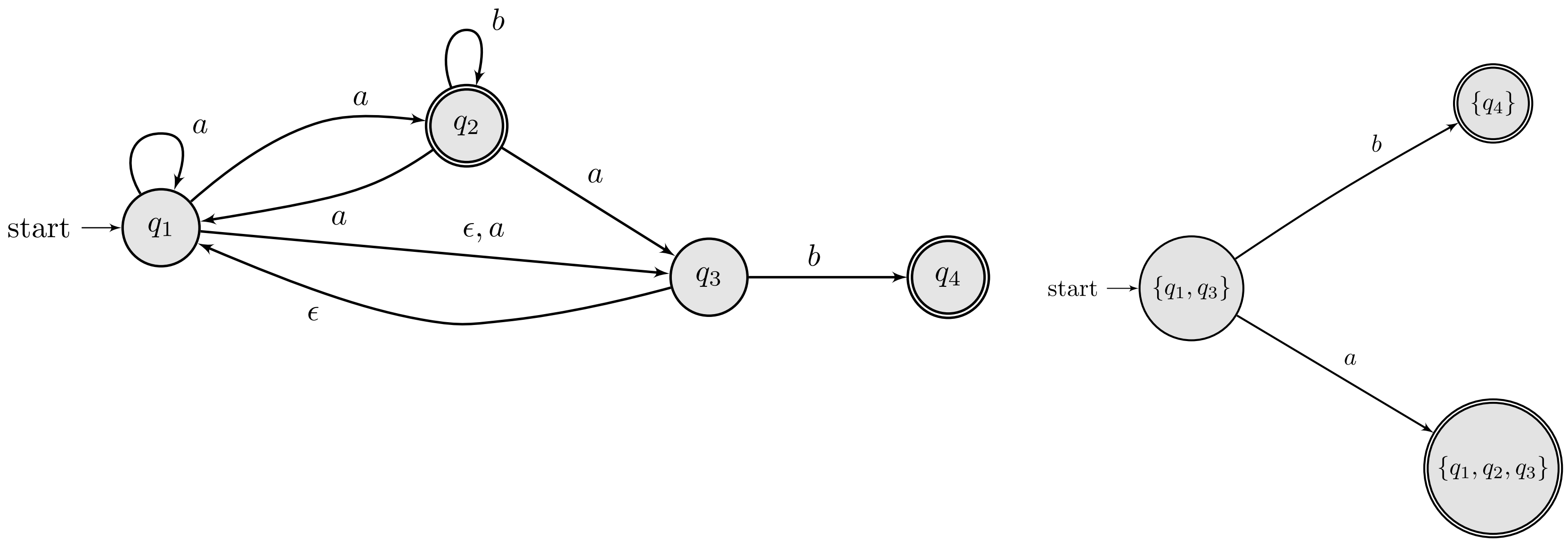
Producing a DFA from an NFA

Producing a DFA from an NFA



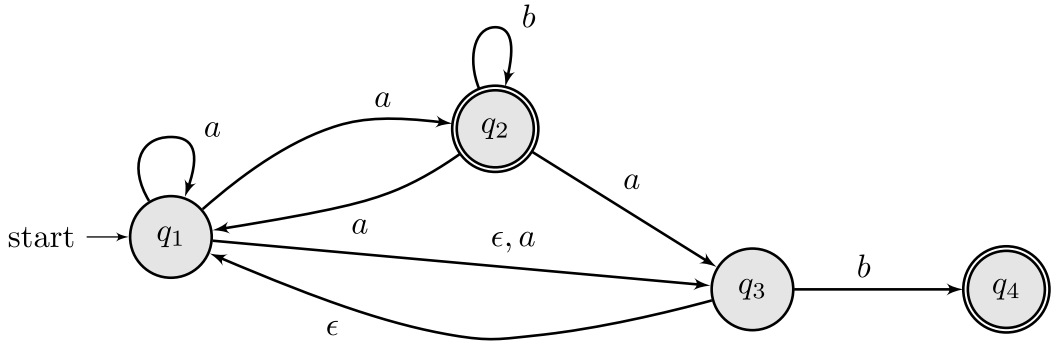
The initial state is the set of all states in the NFA that are reachable from q_1 via ϵ transitions plus q_1.

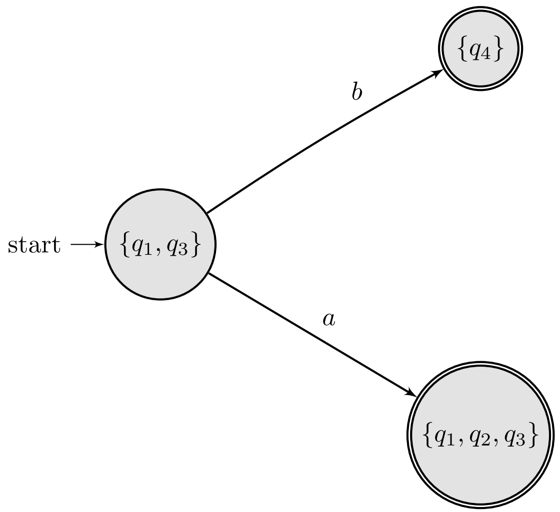
Producing a DFA from an NFA



- For each input in Σ range we must draw a transition to a target state.
- A target state is found by taking an input, say a, and doing an input+epsilon step on each sub-state.

Producing a DFA from an NFA

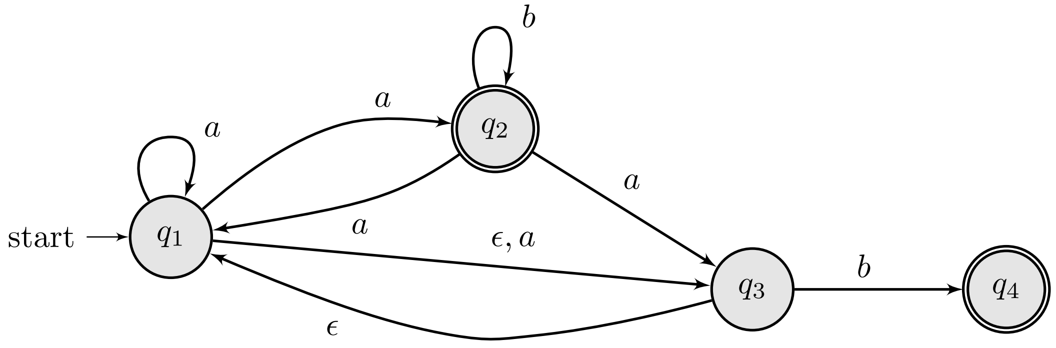


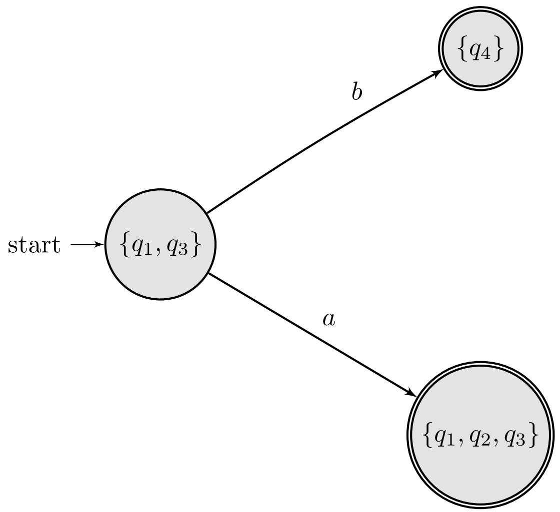


First, input a we find all reachable states (via input+epsilon state) that start from either q_1 or q_3.

- From q_1 via a we get $\{q_1, q_2, q_3\}$
- From q_3 via a we get $\emptyset$
- Result state is $\{q_1, q_2, q_3\} \cup \emptyset = \{q_1, q_2, q_3\}$

Producing a DFA from an NFA

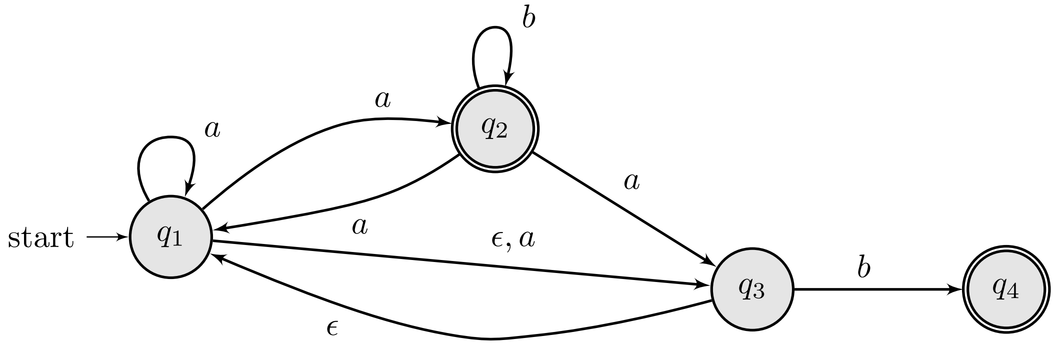


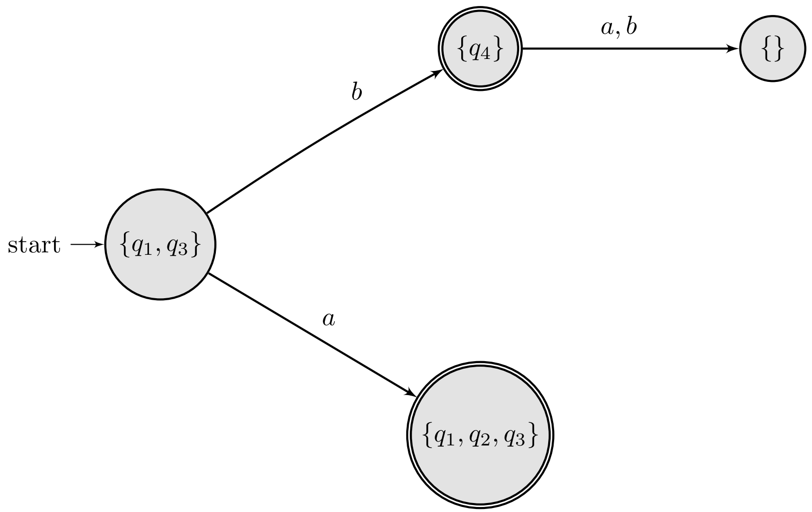


Second, input b we find all reachable states (via input+epsilon state) that start from either q_1 or q_3.

- From q_1 via b we get $\emptyset$
- From q_3 via b we get $\{q_4\}$
- Result state is $\emptyset \cup \{q_4\} = \{q_4\}$

Producing a DFA from an NFA

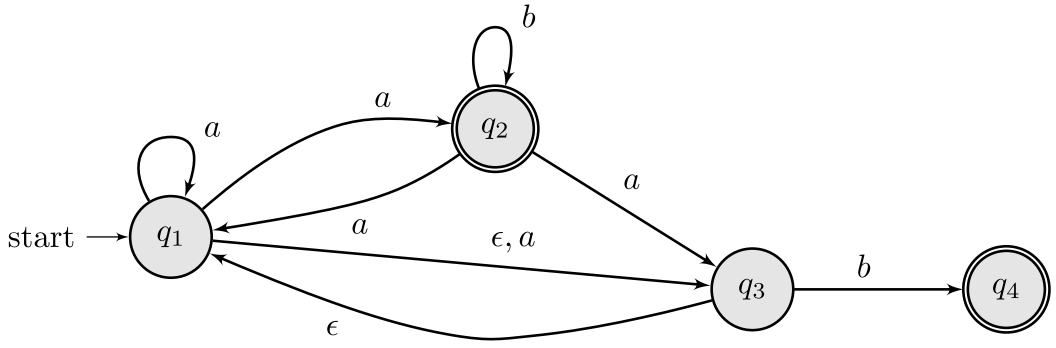


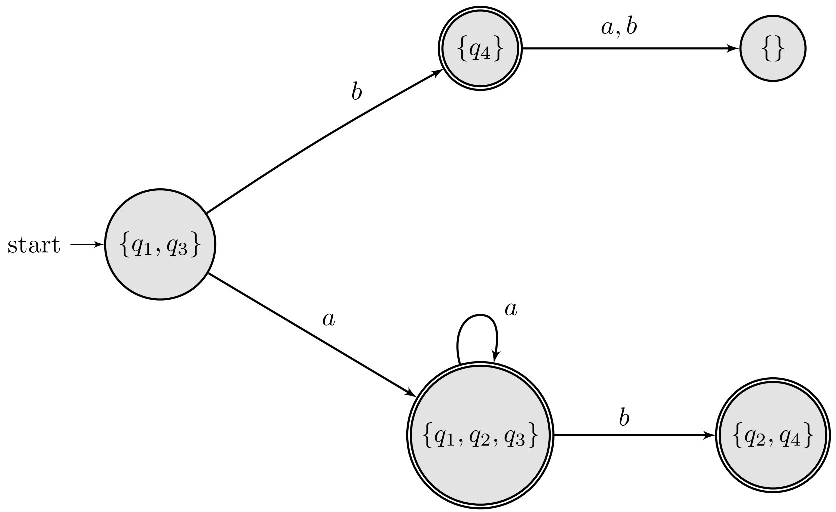


For inputs a and b we find all reachable states (via input+epsilon state) that start from q_4:

- From q_4 via a we get $\emptyset$, so the result state is $\emptyset$
- From q_4 via b we get $\emptyset$, so the result state is $\emptyset$

Producing a DFA from an NFA

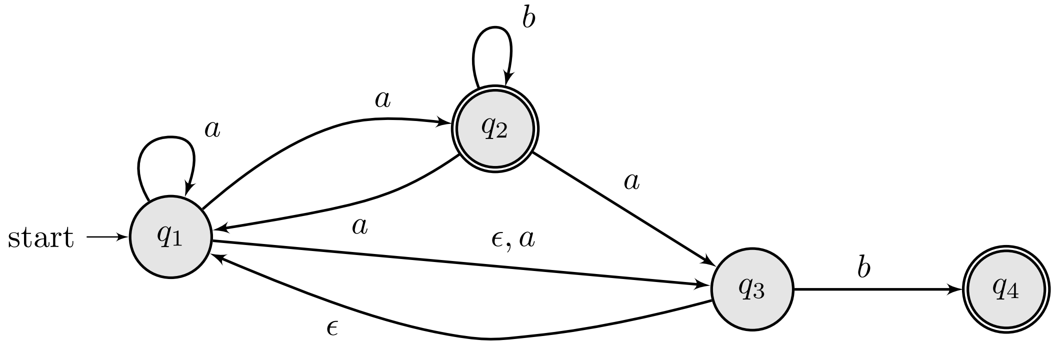


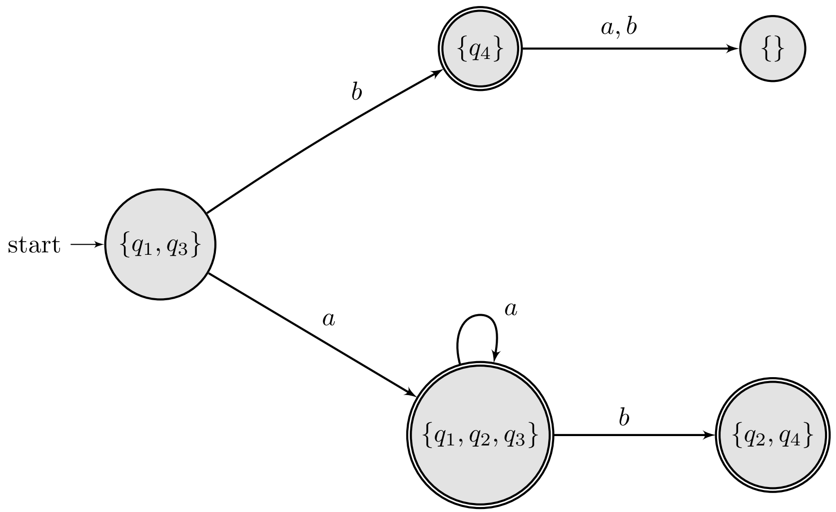


Transition from $\{q_1, q_2, q_3\}$ via a?

- We know with $\{q_1, q_3\}$ with a we reach $\{q_1, q_2, q_3\}$
- From q_2 with a we reach $\{q_3\}$
- Thus, result state is $\{q_1, q_2, q_3\} \cup \{q_3\} = \{q_1, q_2, q_3\}$ (self-loop)

Producing a DFA from an NFA

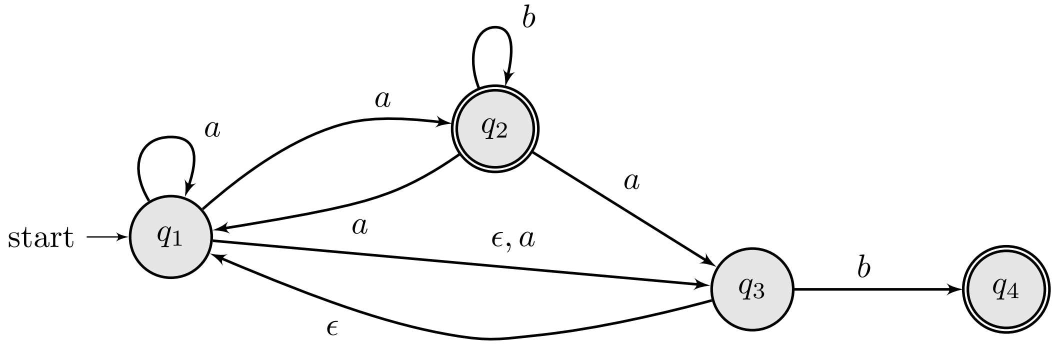


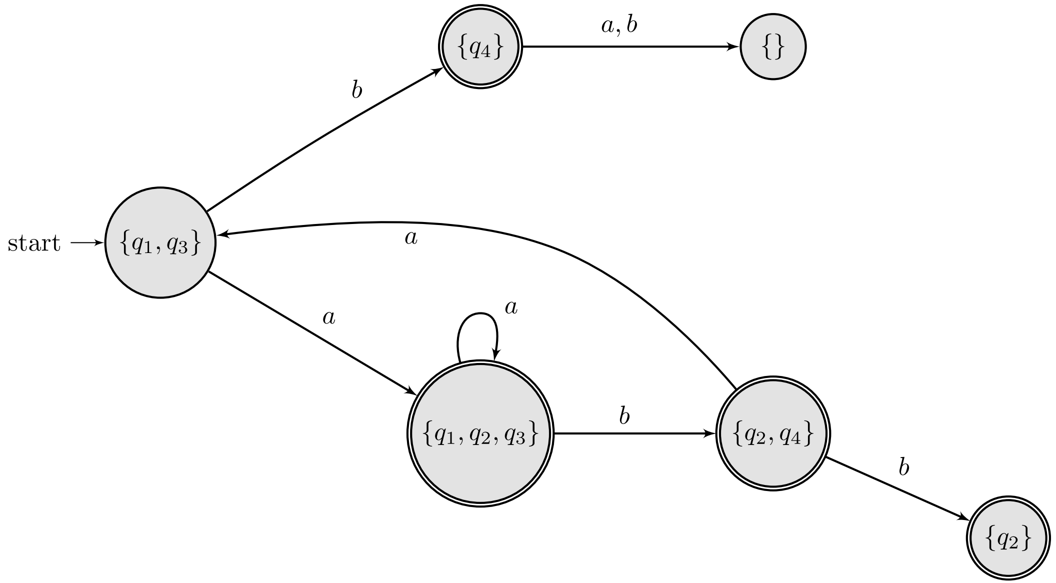


Transition from $\{q_1, q_2, q_3\}$ via b?

- We know with $\{q_1, q_3\}$ with b we reach $\{q_4\}$
- From q_2 with b we reach $\{q_2\}$
- Thus, result state is $\{q_4\} \cup \{q_2\} = \{q_2, q_4\}$

Producing a DFA from an NFA

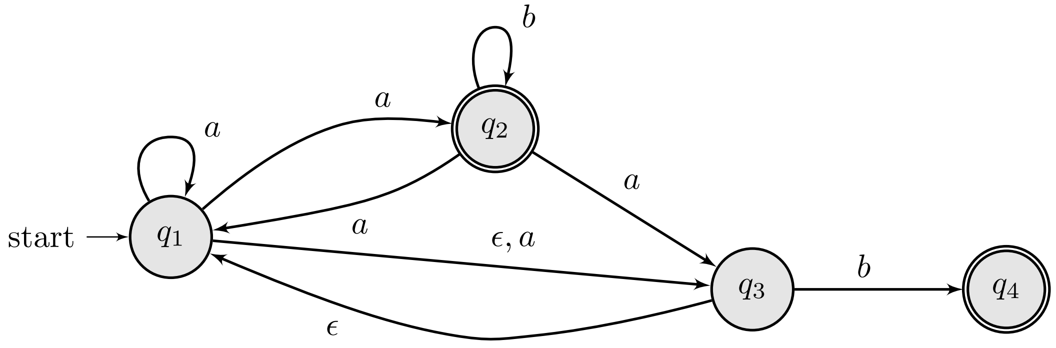


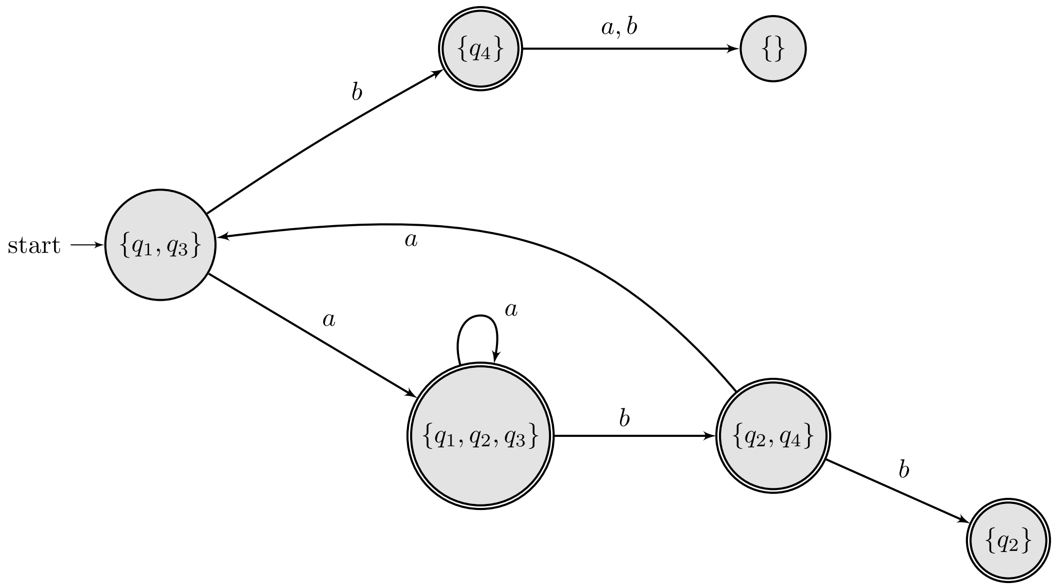


Transition from $\{q_2, q_4\}$ via a?

- From q_2 with a we reach $\{q_1, q_3\}$
- From q_4 with a we reach $\emptyset$
- Thus, result state is $\{q_1, q_3\} \cup \emptyset = \{q_1, q_3\}$

Producing a DFA from an NFA

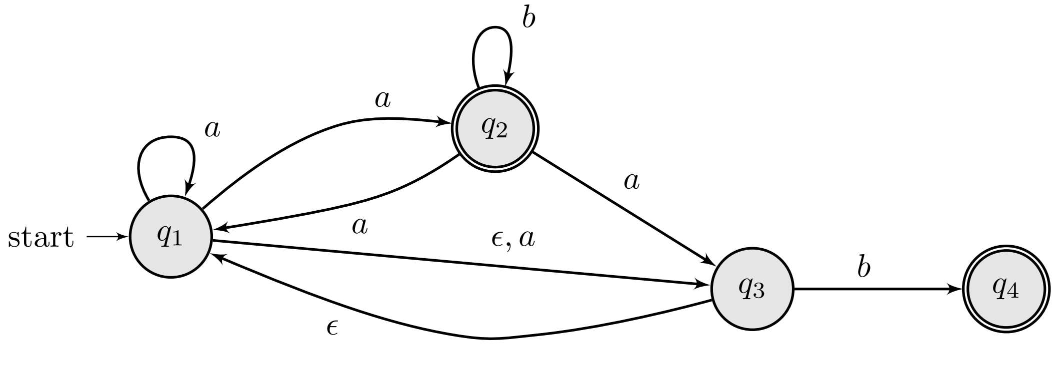


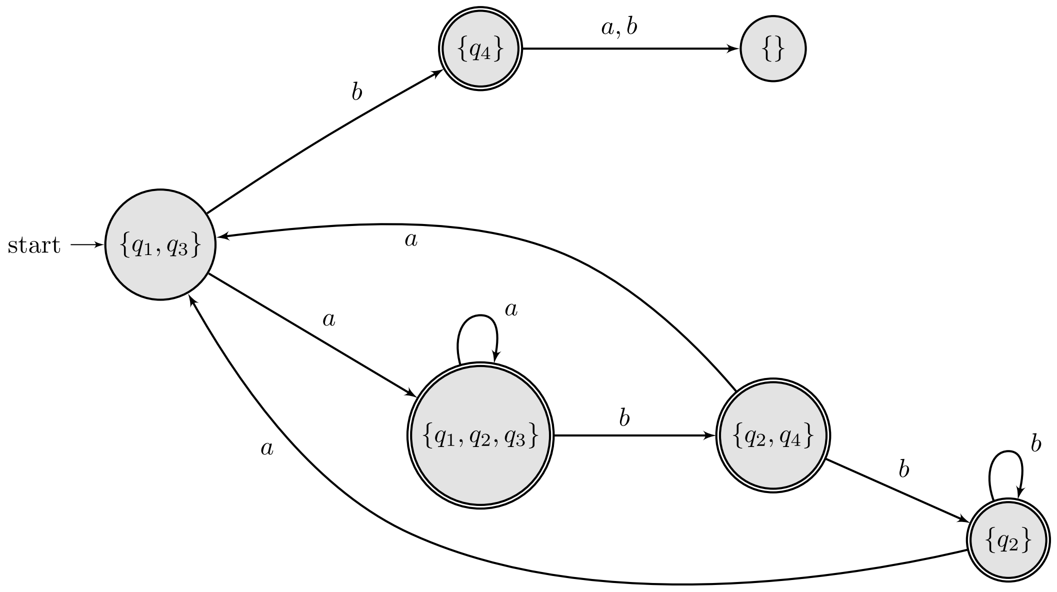


Transition from $\{q_2, q_4\}$ via b?

- From q_2 with b we reach $\{q_2\}$
- From q_4 with b we reach $\emptyset$
- Thus, result state is $\{q_2\} \cup \emptyset = \{q_2\}$

Producing a DFA from an NFA

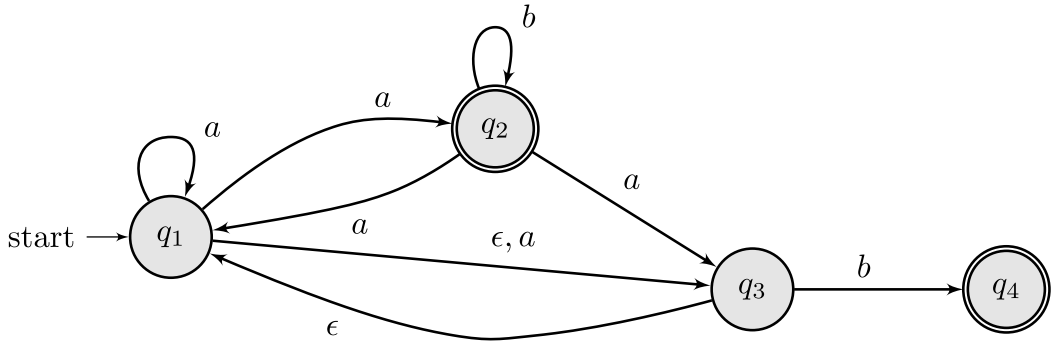


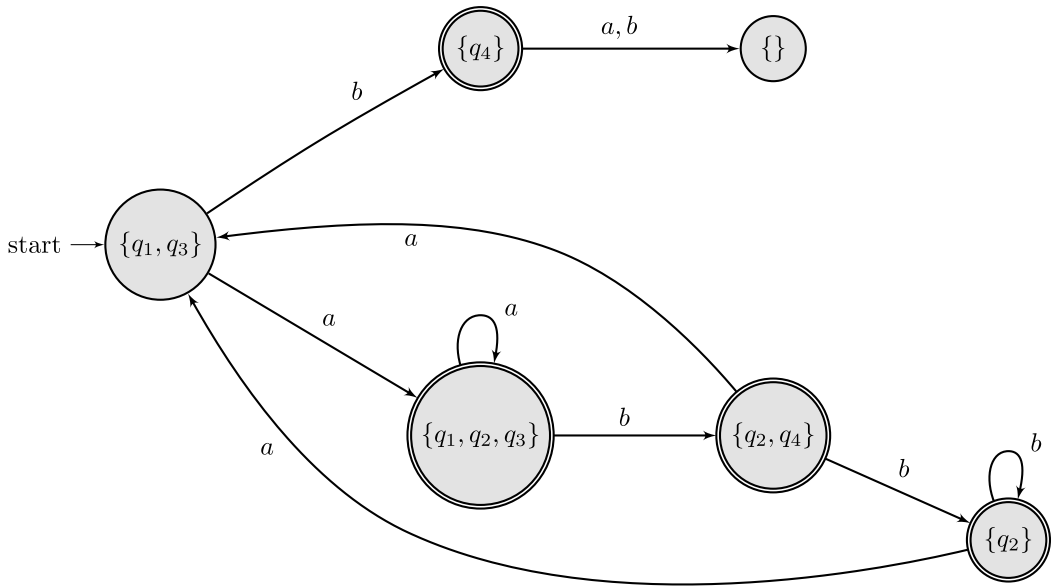


Transition from $\{q_2\}$ via a?

- From q_2 with a we reach $\{q_1, q_3\}$ (result state)

Producing a DFA from an NFA

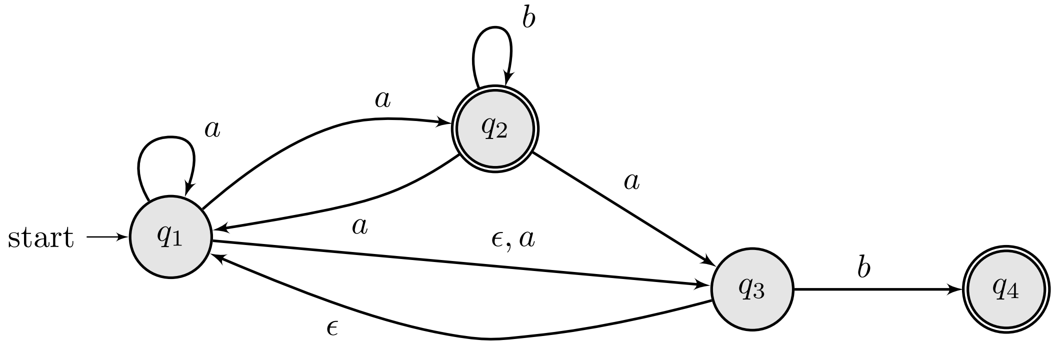


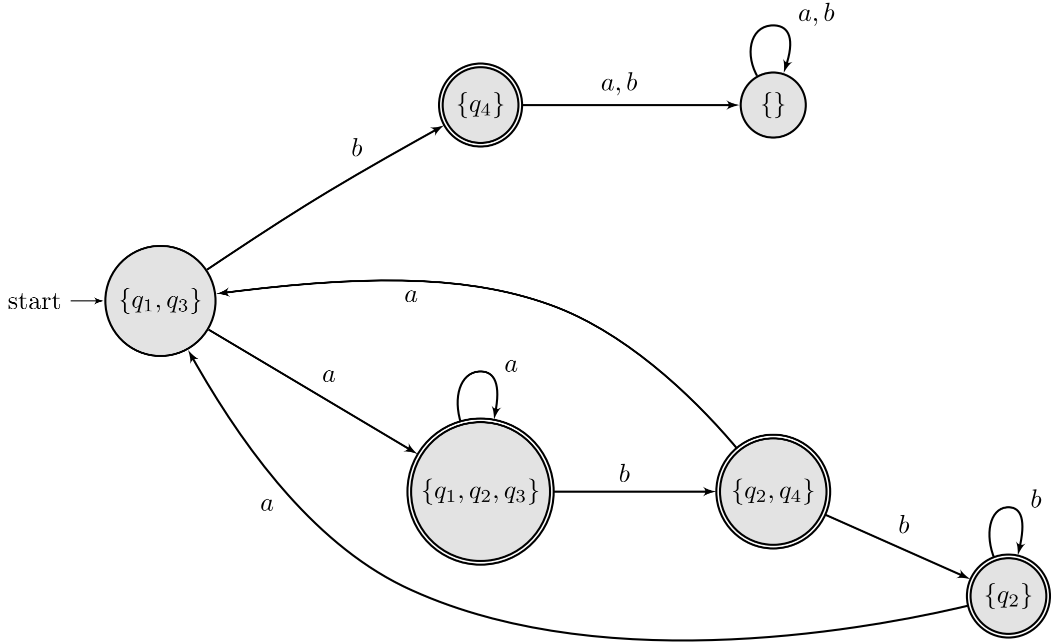


Transition from $\{q_2\}$ via b?

- From q_2 with b we reach $\{q_2\}$ (result state; self loop)

Producing a DFA from an NFA





- State $\{\}$ (also known as $\emptyset$) is a **sink state**, so we draw a self loop for every input in Σ.

Today we will learn...

- Nil
- Empty
- Character
- Union
- Concatenation
- Star

> Section 1.2

The $\mathbf{nil}_\Sigma$ operator

$$L(\mathrm{nil}_\Sigma) = \emptyset$$

The $\mathbf{nil}_\Sigma$ operator

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Example $\Sigma = \{\mathtt{a}, \mathtt{b}\}$

The $\mathbf{nil}_\Sigma$ operator

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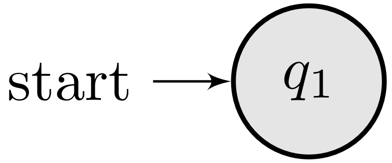

Implementation

```python
def make_nil(alphabet):
  Q1 = 0
  return NFA(
    states=[Q1], alphabet=alphabet, transition_func=lambda q, a: {},
    start_state=Q1, accepted_states=[])
```

The $\mathbf{empty}_\Sigma$ operator

$$L(\mathrm{empty}_\Sigma) = \{\epsilon\}$$

The $\mathbf{empty}_\Sigma$ operator

$$L(\mathbf{empty}_\Sigma) = \{\epsilon\}$$

Example $\Sigma = \{\mathtt{a}, \mathtt{b}\}$

The $\mathbf{empty}_\Sigma$ operator

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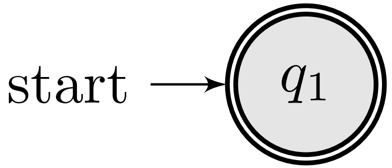
Example $\Sigma = \{\mathtt{a}, \mathtt{b}\}$



Implementation

```
def make_empty(cls, alphabet):
  Q1 = 0
  return NFA(
    states = [Q1],
    alphabet = alphabet,
    transition_func = lambda q, a: {},
    start_state = Q1, accepted_states = [Q1])
```

The $\mathbf{char}_\Sigma(\boldsymbol{c})$ operator

$$L(\mathrm{empty}_\Sigma(c)) = \{[c]\}$$

The $\mathbf{char}_{\Sigma}(c)$ operator

$$L(\text{empty}_{\Sigma}(c)) = \{[c]\}$$

Example $\Sigma = \{\mathtt{a}, \mathtt{b}\}$

The $\mathbf{char}_\Sigma(c)$ operator

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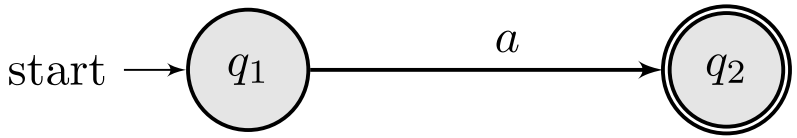
Example $\Sigma = \{\mathtt{a}, \mathtt{b}\}$



Implementation

```python
def make_char(cls, alphabet, char):
  states = Q1, Q2 = 0, 1
  def transition(q, a):
    return {Q2} if a == char and q == Q1 else {}
  return cls(states, alphabet, transition, Q1, [Q2])
```

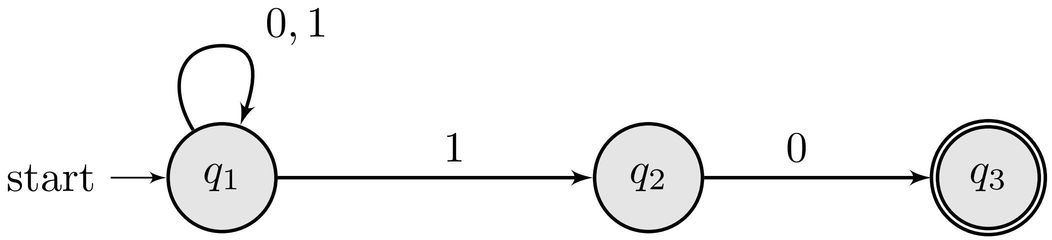
The $\mathbf{union}(M, N)$ operator

$$L(\mathbf{union}(M, N)) = L(M) \cup L(N)$$

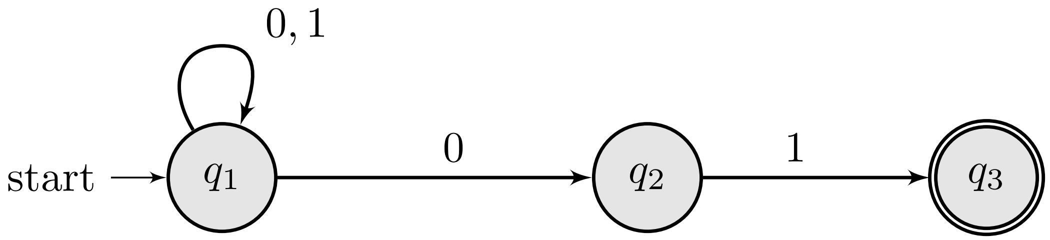
The $\mathbf{union}(M, N)$ operator

$$L(\mathbf{union}(M, N)) = L(M) \cup L(N)$$

N_1



N_2

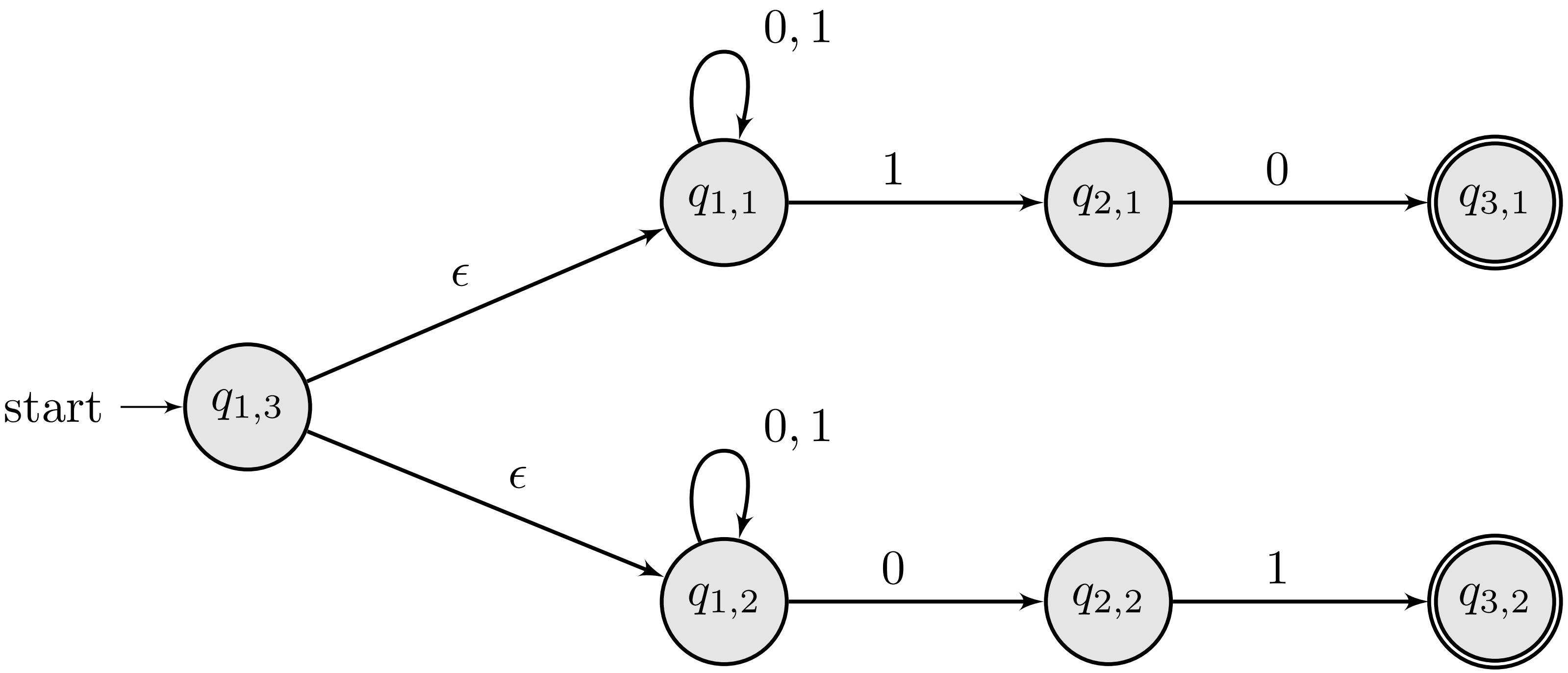


$$\mathbf{union}(N_1, N_2) = ?$$

The $\mathbf{union}(M, N)$ operator

$L(\mathbf{union}(M, N)) = L(M) \cup L(N)$

Example $\mathbf{union}(N_1, N_2)$



- Add a new initial state
- Connect new initial state to the initial states of N_1 and N_2 via ϵ-transitions.

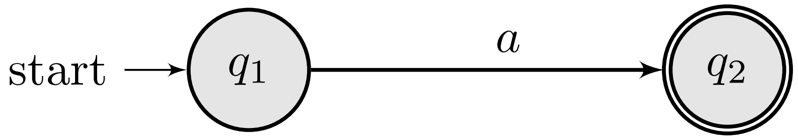
The $\mathbf{concat}(M, N)$ operator

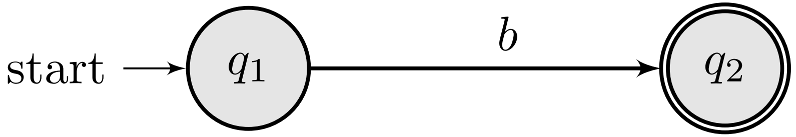
$$L(\text{concat}(M, N)) = L(M) \cdot L(N)$$

The $\textbf{concat}(M, N)$ operator

$$L(\textbf{concat}(M, N)) = L(M) \cdot L(N)$$

Example 1: $L(\textbf{concat}(\textbf{char}(\texttt{a}), \textbf{char}(\texttt{b}))) = \{\texttt{ab}\}$





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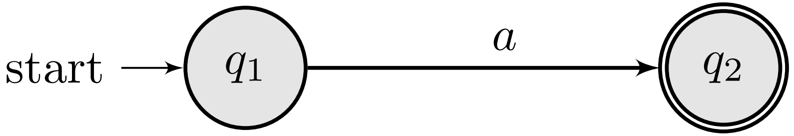
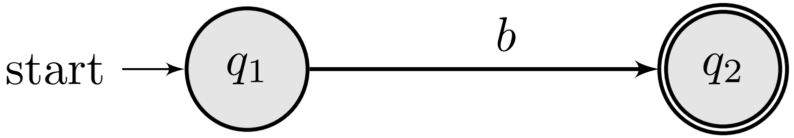
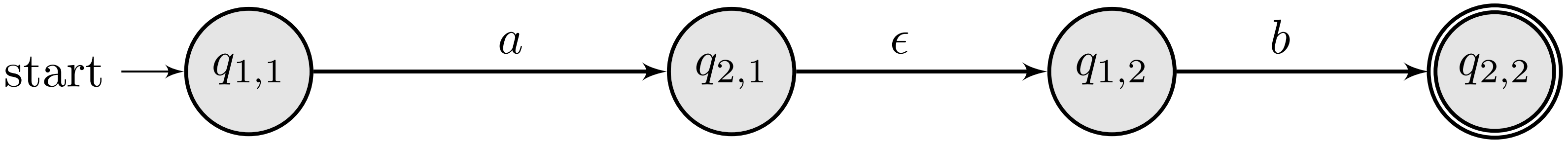




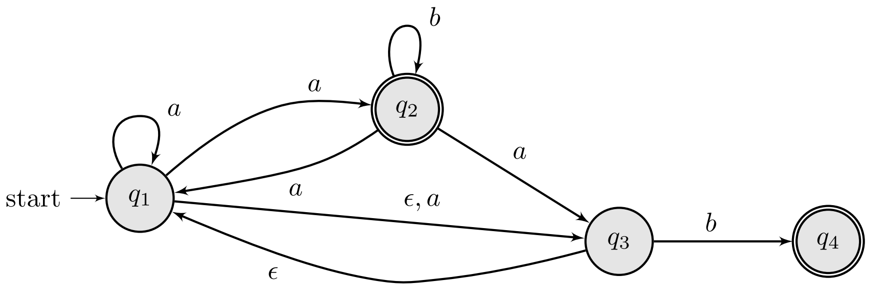
Solution

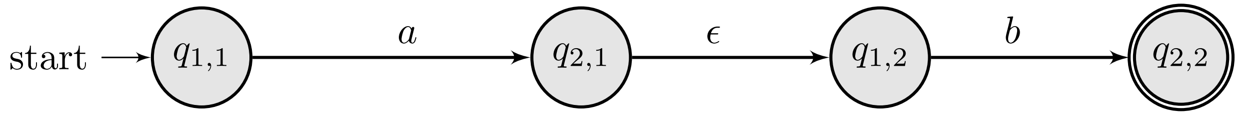


What did we do? Connect the accepted states of N_1 to the initial state of N_2 via ϵ-transitions.

> Why bot connect directly from $q_{1,1}$ into $q_{1,2}$? See next slide.

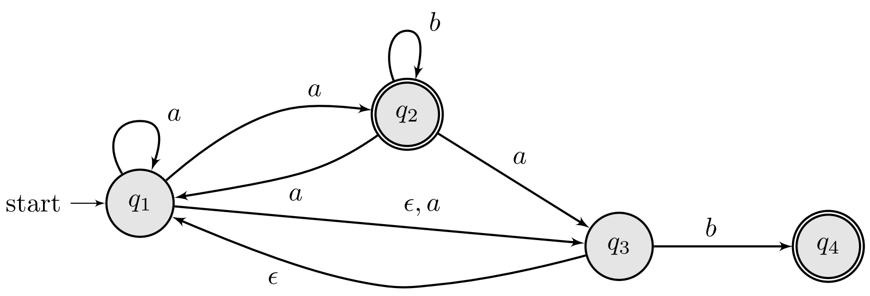
Concatennation example 2

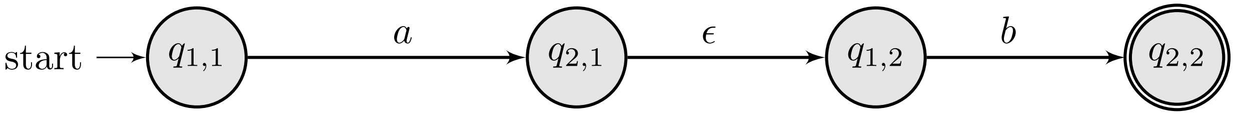




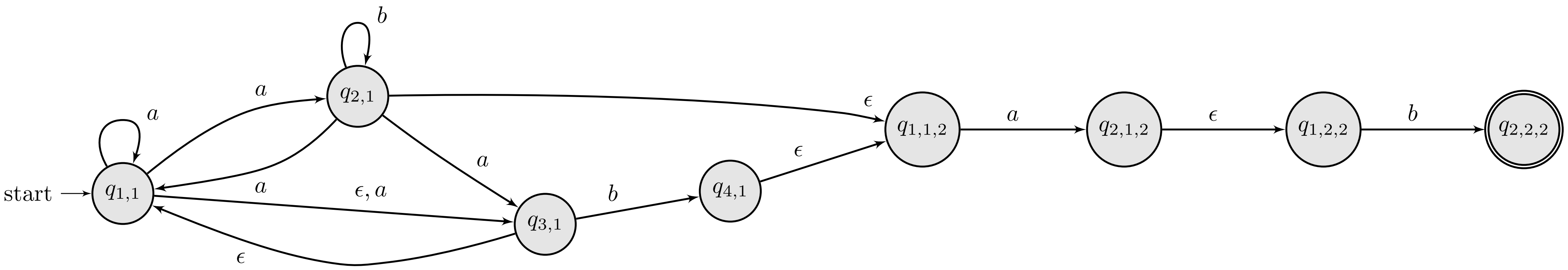
Solution

Concatennation example 2





Solution



Concatenate two NFAs

Let $N_1 = (Q_1, \Sigma_1, \delta_1, q_1, F_1), N_2 = (Q_2, \Sigma_2, \delta_2, q_2, F_2), \mathtt{tag}(Q, n) = \{q^n \mid q \in Q\}$.

We have that $N_1 \cdot N_2 = (Q, \Sigma, \delta, q_1^1, F_2)$ where:

- $Q = \mathtt{tag}(Q_1, 1) \cup \mathtt{tag}(Q_2, 2)$
- $\Sigma = \Sigma_1 \cup \Sigma_2$
- $\delta(q^1, \epsilon) = \{q_2^2\}$ if $q \in F_1$ (*Note: q_2^2 represents the starting state of N_2 tagged with 2.*)
- $\delta(q^n, a) = \mathtt{tag}(\delta_n(r, a), n)$ if $n \in \{1, 2\}$

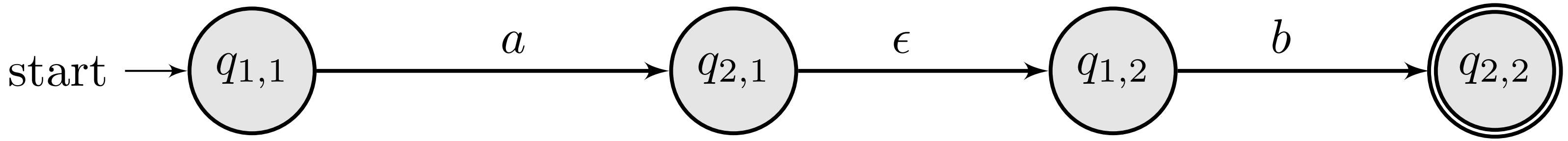
The $\mathbf{star}(N)$ operator

$$L(\text{star}(N)) = L(N)^\star$$

The $\mathbf{star}(N)$ operator

$L(\mathsf{star}(N)) = L(N)^\star$

Example: $L(\mathsf{star}(\mathsf{concat}(\mathsf{char}(\mathtt{a}), \mathsf{char}(\mathtt{b})))) = \{w \mid w \text{ is a sequence of } \mathtt{ab} \text{ or empty}\}$



Solution

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$L(\mathbf{star}(N)) = L(N)^\star$

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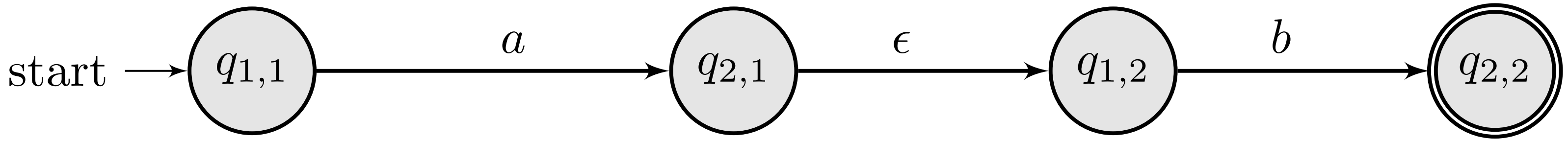
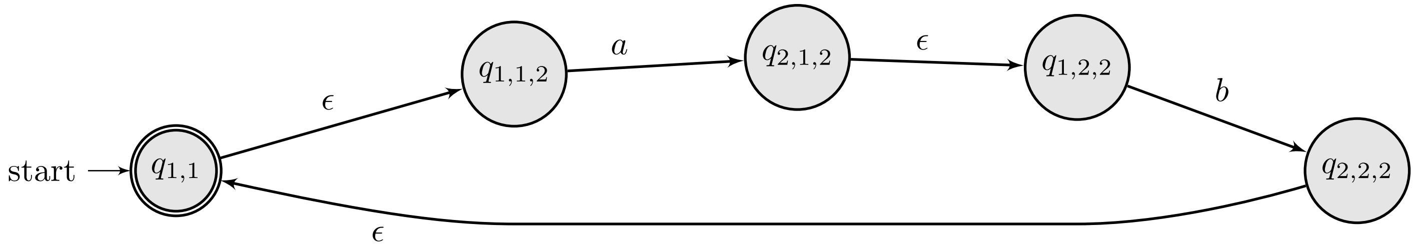


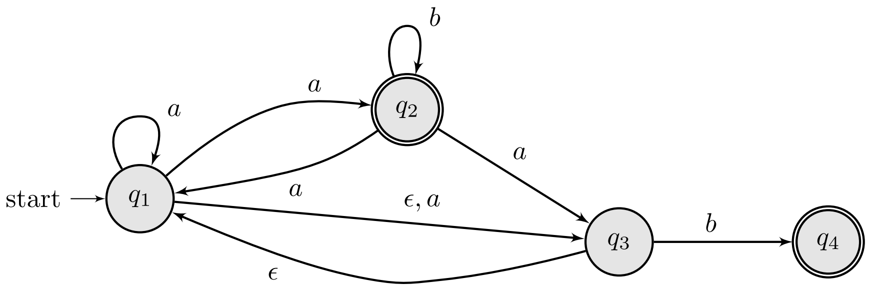
Solution



- create a new state $q_{1,1}$
- ϵ-transitions from $q_{1,1}$ to initial state
- ϵ-transitions from accepted states to $q_{1,1}$
- $q_{1,1}$ is the only accepted state

The $\mathbf{star}(N)$ operator

$$L(\mathbf{star}(N)) = L(N)^\star$$



The $\mathbf{star}(N)$ operator

$$L(\mathbf{star}(N)) = L(N)^{\star}$$