Homework Assignment 6

Suggested textbook Theorems: 1.45, 1.47, 1.49, 2.32. You can also assume the results in Exercise 2.16 and that context-free languages are Turing recognizable.

1. Prove or disprove the following statements. Your answer should have two parts: (i) wether or not the statement holds; (ii) a formal proof that backs your claim.
   (a) If $L_1 \subseteq L_2$ and L_2 is regular, then L_1 is regular.
   (b) If L_1 is regular and L_2 is context-free, then $L_1 \cup L_2$ is context-free.
   (c) If $L_1 \cdot L_2$ is not context free, then either L_1 is not context free or L_2 is not context free.
   (d) If L is context-free, then L is regular.
   (e) If $L^\star$ is not context free, then L is not context free.
   (f) If L is not regular, then L is not context-free.
   (g) For any language L_1 there exists a regular language L_2 such that $L_2 \subseteq L_1$.
   (h) A Turing machine can write the blank character $\textvisiblespace$ in the tape.
   (i) In a Turing machine, the input Σ and the tape alphabet Γ must differ.
   (j) A Turing machine contains at least 3 different states.
2. Given that $\{a^n b^n c^{n+1} \mid n \geq 0\}$ is not context-free and without using the Pumping Lemma or the Theorem of Non-Context-Free Languages from Lecture 11, show that $\{a^n b^n c^n \mid n \geq 0\}$ is not context-free.
3. Let M be the Turing Machine of Example 2 in Lecture 12. Show the configuration history that rejects string `001`.
4. Consider language $L_1 = \{w\#w \mid w \in \{\mathtt{a}, \mathtt{b}\}^\star\}$. Check all that apply.
   - ◯ regular
   - ◯ not regular
   - ◯ context free
   - ◯ not context-free
   - ◯ Turing recognizable